Notched Hollow Square Section Steel Column Buckling

Zia Razzaq, and Solomon Tecleab

Abstract—Presented in this paper is an outcome of a study to assess the effect of section loss in the form of longitudinal notches on the buckling load of hollow square section steel columns. The theoretical study includes buckling load estimates based on both an iterative equilibrium as well as a non-iterative energy approach. Buckling loads based on sample laboratory experiments are also presented. The study shows that the presence of a notch can significantly reduce the axial load-carrying capacity of a steel column.

Index Terms—Buckling Load; Steel Columns; Notched Hollow Square Section; Equilibrium and Energy Approaches.

I. INTRODUCTION

A steel column can experience a partial loss of section due to corrosion or other causes which can reduce its buckling load capacity. Even a small sectional loss in localized zones may lead to premature structural failure. For such a column, strengthening or member replacement may be warranted, however, it is necessary to first assess its residual load-carrying capacity and then decide the options of replacement or retrofitting. Only a limited number of studies have been published in the past on the performance of columns with section loss with none related to notched hollow square or rectangular sections.


Liu and Young [8] presented the outcome of laboratory tests on cold-formed steel square hollow section members with fixed ends and subjected to axial compression and compared the results with formulas in design specifications from USA, Australia, New Zealand, and Europe. The present paper is focused on an elastic buckling load study of notched hollow square section steel columns. Solutions based on the governing differential equations in both un-notched and notched regions are formulated and then solved for the buckling load. The effect of the notch size on the column load-carrying capacity is determined. A comparison of the results is made to those obtained using an energy approach. Sample laboratory experimental results are also presented.

II. PROBLEM DEFINITION

Figure 1 shows a steel column with un-notched and notched cross sections. The column is subjected to a gradually increasing axial load P until buckling occurs. The figure also shows various dimensions defining the geometry of the column, its buckled shape, and the sections in regions with no notch and those with a symmetrically located notch of length “a” and width (gap) W. Figure 2 is a schematic of the column and the notch shape. Figure 3 shows a typical notched column buckling test setup.

![Fig. 1. Column with un-notched and notched sections.](image-url)

The problem addressed herein is to determine the influence of a notch on the buckling load of the steel column using both equilibrium and energy approaches, and compare the theoretical results with those obtained experimentally for the equilibrium approach is based on governing differential equations for the problem whereas the energy formulation utilizes an approximate but fairly accurate expression for the buckled shape of the column.
III. THEORETICAL ANALYSIS

A. Equilibrium Approach:

Modifying the governing differential equation for a uniform column [7], the differential equations for each portion of the notched column shown in Figure 1 are formulated as follows:

\[ EI_1 \frac{d^2 y_1}{dx^2} = P(\delta - y_1), \quad 0 \leq x \leq \frac{a}{2} \]

(1)

\[ EI_2 \frac{d^2 y_2}{dx^2} = P(\delta - y_2), \quad \frac{a}{2} \leq x \leq \frac{L}{2} \]

(2)

The solutions to (1) and (2) are found to be as follows:

\[ y_1 = \delta + Acos k_1 x + Bsin k_1 x \]

(3)

\[ y_2 = \delta + Ccos k_2 x + Dsin k_2 x \]

(4)

in which:

\[ k_1 = \sqrt{\frac{P}{EI_1}} \quad \text{and} \quad k_2 = \sqrt{\frac{P}{EI_2}} \]

The boundary conditions at \( x = 0 \) are given by:

\[ y_1 = 0; \quad \frac{dy_1}{dx} = 0 \]

(5)

The following condition applies at \( x = \frac{L}{2} \):

\[ y_2 = \delta \]

(6)

The continuity conditions at \( x = \frac{a}{2} \) between the notched and un-notched regions are as follows:

\[ y_2 = y_1; \quad \frac{dy_1}{dx} = \frac{dy_2}{dx} \]

(7)

Substituting (5)-(7) into (3) and (4) finally results in the following transcendental expression:

\[ \frac{k_2}{k_1} = \tan k_1 \frac{a}{2} \tan k_2 \frac{L}{2} \]

(8)

Equation (8) can be iteratively solved for the buckling load \( P \) if the moment of inertia ratio \( \frac{I_1}{I_2} \) and \( \frac{a}{L} \) are known.
B. Energy Approach:

The buckled shape of the column shown in Figure 1 can be approximated by the following expression:

\[ y = \delta(1 - \cos \frac{x}{L}) \]  

(9)

The flexural strain energy of the buckled curve is given by [7]:

\[ \Delta U = 2 \left[ \frac{a}{2} \frac{M^2}{2EI_1} dx + \frac{L}{2} \frac{M^2}{2EI_2} dx \right] \]  

(10)

The bending moment M at any point along the column length is given by:

\[ M = -P(\delta - y) \]  

(11)

The work done by P due to the vertical displacement of the column is given by:

\[ W_{ext} = \frac{P}{2} \int_0^L \left( \frac{dy}{dx} \right)^2 dx = \frac{P\pi^2\delta^2}{4L} \]  

(12)

Using (9) and (11) in (10) and then equating its right-hand side with that of (12) results in the following buckling load expression:

\[ P_{cr} = \frac{\pi^2EI_2}{L^2} m \]  

(13)

where m is given by:

\[ m = \left[ \frac{1}{1 + \frac{a}{L} \frac{I_2}{I_1} (\frac{L}{l} - 1)} + \frac{1}{\pi} \sin \frac{\pi a}{L} \frac{I_2}{I_1} (\frac{L}{l} - 1) \right] \]  

(14)

Table I summarizes numerical results for un-notched Column No. 1 and notched Column Nos. 2 through 13 based on (8) and (13) corresponding, respectively, to the iterative equilibrium, and non-iterative energy approach. The results in this table are generated using the following input data: column length L equal to 47.875 in.; 1 x 1 x 0.125 in. cross section; and a Young’s modulus value E of 29,800 ksi. Furthermore, a and W are the respective length and width of the notch; I1/I2 is the ratio of the moment of inertia of the cross section in the notched and un-notched regions; PzA and PzB are the column buckling load values based on the equilibrium and energy approaches, respectively; and the ratio p equals PzA/PzB. ratio of the moment as seen from the p values, the results from the equilibrium and energy approaches are in excellent agreement.

<table>
<thead>
<tr>
<th>No.</th>
<th>a/L</th>
<th>W (in.)</th>
<th>I1/I2</th>
<th>PzA (kips)</th>
<th>PzB (kips)</th>
<th>Ratio p</th>
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<tr>
<td>1</td>
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<td>0.000</td>
<td>1.000</td>
<td>7.310</td>
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<tr>
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<td>0.125</td>
<td>0.894</td>
<td>7.005</td>
<td>7.147</td>
<td>0.980</td>
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<td>3</td>
<td>0.376</td>
<td>0.125</td>
<td>0.894</td>
<td>6.768</td>
<td>6.991</td>
<td>0.968</td>
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<tr>
<td>4</td>
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<td>0.125</td>
<td>0.894</td>
<td>6.619</td>
<td>6.842</td>
<td>0.967</td>
</tr>
<tr>
<td>5</td>
<td>0.752</td>
<td>0.125</td>
<td>0.894</td>
<td>6.549</td>
<td>6.699</td>
<td>0.978</td>
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<tr>
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<td>6.953</td>
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<tr>
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<tr>
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<td>0.578</td>
<td>5.712</td>
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<tr>
<td>12</td>
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<td>5.149</td>
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<tr>
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<td>0.578</td>
<td>4.687</td>
<td>4.687</td>
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The results in Table I indicate that the column buckling load decreases with an increase in the notch dimensions, namely, notch length a and its width W. For example, the buckling load of the notched Column No. 13 is about 64 percent of the un-notched Column No. 1. However, these results are based on the elastic theory and under the condition that the notch width W does not change due to deformations caused by the applied axial load.

IV. EXPERIMENTAL STUDY

Using the test setup shown in Figure 3, experiments were conducted on Column Nos. 1 through 3, and 11. For the ‘baseline’ un-notched Column No. 1, the ratio of the theoretical buckling load to that observed experimentally was found to be about 1.10. The same ratio was observed for the notched Column No. 2. The 10 percent higher theoretical load values for these two columns are indicative of possible inelastic action coupled with any manufacturing residual stresses in the test specimens. For Column No 3, the theoretical to experimental load ratio was found to be 1.18. For Column No. 11, it was observed during the loading process that the notch ‘gap’ W started to become narrower at and around the column mid-height as the applied load P was increased until the gap was completely eliminated in the said region. The column failed at about one-third of its expected capacity due to this cross-sectional distortion coupled with inelastic action at and around the notched region. It was therefore concluded that the predicted buckling loads would be in good agreement with the experimentally observed ones as long as the notch width W did not change with an increase in the axial load up to the collapse condition. The experimental results for the columns tested warrant a further evolution of the theoretical prediction model that includes any attendant cross-sectional distortions and inelastic behavior.

V. CONCLUSION

An iterative theoretical equilibrium approach for predicting the buckling load of notched hollow section steel columns showed an excellent agreement with that based on a non-iterative energy formulation. The prediction models can provide approximate estimates of the column buckling...
load as long as its behavior remains within the elastic range and without the development of cross-sectional distortions. Further evolution of the prediction models should include the effects of inelastic action and any possible cross-sectional distortions as the applied axial load is increased up to the collapse condition.

REFERENCES


Zia Razzaq received a Doctor of Science degree in civil engineering from Washington University St. Louis, Missouri, USA in 1974. He is currently a University Professor in the Department of Civil and Environmental Engineering, Old Dominion University, Norfolk, Virginia 23529, USA.

Dr. Razzaq is a registered Professional Engineer in the Commonwealth of Virginia, USA, and was elected Fellow of the American Society of Civil Engineers in 1988.