Cardinal Points Symmetry Landmarks Distribution Model for Segmentation of Region of Interest in Simulated B-Mode Ultrasound Carotid Artery Images

Charles Nnamdi Udekwe, and Akinlolu Adediran Ponnle

Abstract—Measurement accuracy and understanding of geometry of Common Carotid Artery (CCA) play an important role in carotid atherosclerosis assessment and management. In order to devise techniques that can accurately analyze the effects of plaques on the carotid artery, a model that can sufficiently segment the Region of Interest (ROI) in the B-mode ultrasound image of carotid artery is needed. In this paper, a new Cardinal Points Symmetry Landmarks Distribution Model (CPS-LDM) to sufficiently segment the ROI in the carotid artery B-mode ultrasound imaged in the transverse plane is developed. The developed model employs a combination of fixed landmarks (FLs) and movable landmarks (MLs) to obtain the total landmarks (TLs) that can sufficiently segment the shape of the ROI of the carotid artery. Simulated ultrasound images are used. Four FLs are fixed on each of the four ROIs of the simulated carotid artery determined by the cardinal points North (N), South (S), East (E) and West (W) drawn on the ROIs of the carotid artery. The MLs are determined by the inter-cardinal directions such as North-East (NE), North-West (NW) and so on. The CPS-LDM equation developed allows us to visualize graphically the optimum number of points that can sufficiently segment the ROIs. ImageJ2 software was used to generate the Cartesian coordinates for each landmark which were then used to generate the Shape Space Pattern (SSP) of the carotid artery ready for further statistical analysis. The results showed that the CPS-LDM model is sufficiently generic and adaptable to a variety of carotid artery B-mode ultrasound image simulated under different scenarios.

Index Terms—Cardinal Points, Carotid Artery, Landmarks, Region of Interest, Ultrasound.

I. INTRODUCTION

One area in which ultrasonography has found very useful application in the medical field is in the area of cardiovascular image diagnosis. Recently, the increase in the number of patients suffering from cardiovascular diseases has become a serious social problem [1]. The most well-known risk factor for the development of cardiovascular events is internal carotid artery stenosis also known as internal carotid artery disease [2]. Carotid stenosis is the constriction of the lumen of the carotid artery as a result of Carotid Atherosclerosis [3]. Carotid atherosclerosis is a condition in which the carotid artery wall thickens as a result of plaque formation. Plaques are made up of cholesterol and fatty acids, calcium, and sometimes fibrous connective tissue [4]. Stroke is one of the leading causes of long-term disability and death throughout the world. It is caused by the disruption of brain function due to ischemia, and carotid atherosclerosis is one of the major causes of ischemia [5], [6], [7]. Most strokes are caused by a blood clot, plaque buildup, or a combination of the two which are known as ischemic strokes. Around 87% of all strokes are ischemic strokes [8]. Internal carotid artery stenosis has been found to be responsible for 30% of ischemic strokes [9] and it has also been shown to be a strong predictor of death in the general population [10], [11].

Measurement accuracy and understanding of geometry of Common Carotid Artery (CCA) play an important role in carotid atherosclerosis assessment and management [12]. The typical arterial wall consists of three layers: an innermost layer, “the intima”, a middle layer, “the media”, and an outer layer, “the adventitia” [13]. This is illustrated in Fig. 1.

Fig. 1. A schematic cross section of an artery. From within, the arterial wall consists of three layers: intima, media and adventitia [13].

Important factors in diagnosis of atherosclerotic disease of the carotid arteries are the intima-media thickness (IMT), plaque morphology, criteria for grading stenosis, limiting factors such as the presence of dissection or cardiac abnormalities, distinction between near occlusion and total occlusion, and the presence of a subclavian steal [14]. The IMT measurement is indicative of the thickness of the arterial wall, and is precisely imaged using ultrasound technology [15]. This imaging can be in two planes: the longitudinal plane (long axis plane) and the transverse plane (short axis plane).

In this paper, we developed a novel Cardinal Point Symmetry Landmark Distribution Model (CPS-LDM) for the complete geometric shape characterization of simulated carotid artery B-mode ultrasound images in the transverse plane (short axis plane). The model consists of the following components.

1) Cardinal point and inter-cardinal point symmetry description of the simulated carotid artery.
2) Fixed landmarks (FLs) equation, movable landmarks
(MLs) equation, and the total landmarks (TLs) equation to yield the CPS-LDM equation.

3) Shape Space Pattern (SSP) of the carotid artery based on the CPS-LDM equation developed.

The remainder of this paper is organized as follows. Section II describes the Cardinal Point Symmetry (CPS) of the simulated carotid artery geometry used in deriving the CPS-LDM model. Section III derives the fixed, movable and total landmarks on the CPS carotid artery. Section IV shows the results obtained from the developed equations and conclusions are finally drawn in Section V.

II. DEVELOPMENT OF CARDINAL POINT SYMMETRY (CPS) MODEL FOR SIMULATED CAROTID ARTERY IMAGE

A. Data Acquisition

Images were obtained from [16] which were simulated images using Field II ([17], [18]) a MATLAB based program that can simulate all kinds of ultrasound transducers and other associated ultrasound images. Figure 2 shows a sample of a simulated B-mode ultrasound carotid artery obtained using Field II software. Each landmark coordinate was registered and digitized using ImageJ2 software [19].

![Fig. 2: Simulated B-mode ultrasound image of carotid artery in the transverse plane obtained from [16] using Field II software.](image)

B. CPS Model Description

The four cardinal points also known as cardinal directions are the North, East, South, and West, commonly denoted by their initials N, E, S, and W. The inter-cardinal points or directions are North-East (NE), South-East (SE), South-West (SW), and North-West (NW). Figure 3 shows the cardinal and inter-cardinal points used to develop the CPS-LDM. The cardinal points concept was then used to describe and label strategic points on the carotid artery image which subsequently led to the full description of the carotid artery image. This is shown in Fig. 4 and Fig. 5.

![Fig 3: The cardinal and inter-cardinal points used to develop the CPS-LD Model.](image)

In Fig. 4, an imaginary VS – VS (Vertical Symmetry) line is drawn across the ROIs such that it passes through the midpoint of the adventitia and media boundary of ROI 1, the Media and Intima boundary of ROI 2, the Media and Intima boundary of ROI 3 and the media and adventitia boundary of ROI 4. The point where the VS – VS line touches the midpoints of the four ROIs are marked as NX, SX, where N and S are the cardinal points North and South respectively and the index X are four integers 1, 2, 3, 4, marking the four ROIs. Table 1 shows a detailed description of each label used in Fig. 4.

![Fig. 4: Fixed cardinal points on the simulated carotid artery image.](image)

![Fig. 5: Full labelling and description of the simulated carotid arterial wall image.](image)

In Fig. 5, the VS-VS line also divides the ROIs into two equal halves which are the Eastern (Rightward) half and the Western (leftward) half. The lefmost tip of ROI 1 is marked West1 i.e. W1 and the rightmost tip of ROI 1 is marked East1 i.e. E1. The same designation goes for the other ROIs so that we have W2 and E2 for ROI 2, W3 and E3 for ROI 3 and W4 and E4 for ROI 4. This designation led to the full description of the imaged carotid arterial wall in the transverse plane shown in Fig. 5. Table 2 shows a detailed description of each notation used in Fig. 5.
TABLE I: DESCRIPTIONS OF LABELS IN FIG. 4

<table>
<thead>
<tr>
<th>Label</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>VS – VS</td>
<td>Vertical Symmetry Line. This line divides the Region of Interests (ROIs) into two equal halves; Westward (left), and Eastward (Right).</td>
</tr>
<tr>
<td>HS – HS</td>
<td>Horizontal Symmetry Line. This line divides the ROIs into two regions which are the Anterior Region (AR) and the Posterior Region (PR). The first two ROIs (ROI [1] and ROI [2]) occupy the AR while the last two ROIs (ROI [3] and ROI [4]) occupy the PR.</td>
</tr>
<tr>
<td>QUADRANTS [Q]</td>
<td>The VS – VS and HS – HS lines divide the four ROIs into four Quadrants [Q1], [Q2], [Q3], [Q4]. Each half of ROIs [1] and [2] occupies [Q1] and [Q2] respectively while each half of ROIs [3] and [4] occupies [Q3] and [Q4] respectively.</td>
</tr>
<tr>
<td>ROI [1]</td>
<td>This is the upper anterior (UA). It is defined by the cardinal points West, North, and East, abbreviated as (WNE), on its outer arc. This outer arc is the Adventitia boundary. Its inner arc is defined by the cardinal points West, South, and East, abbreviated as (WSE). This inner arc is known as the Media boundary. Hence, the ROI [1] is bounded by the Media and the Adventitia boundary (MAB).</td>
</tr>
<tr>
<td>ROI [2]</td>
<td>This is the lower Anterior (LA). It is defined by the cardinal points West, North, and East, abbreviated as (WNE), on its outer arc. This outer arc is the Media boundary. Its inner arc is defined by the cardinal points West, South, and East, abbreviated as (WSE). This inner arc is known as the Intima boundary. Hence, the ROI [2] is bounded by the Media and the Intima boundary (MIB).</td>
</tr>
<tr>
<td>ROI [3]</td>
<td>This is the Lower Posterior (LP). It is defined by the cardinal points West, North, and East, abbreviated as (WNE), on its outer arc. This outer arc is the Media boundary. Its inner arc is defined by the cardinal points West, South, and East, abbreviated as (WSE). This inner arc is known as the Intima boundary. Hence, the ROI [3] is bounded by the Media and the Intima boundary (MIB).</td>
</tr>
<tr>
<td>ROI [4]</td>
<td>This is the Upper Posterior (UP). It is defined by the cardinal points West, North, and East, abbreviated as (WNE), on its outer arc. This outer arc is the Adventitia boundary. Its inner arc is defined by the cardinal points West, South, and East, abbreviated as (WSE) on its outer arc. This outer arc is the Adventitia boundary. Its inner arc is defined by the cardinal points West, South, and East, abbreviated as (WSE). This inner arc is known as the Media boundary. Hence, the ROI [4] is bounded by the Media and the Adventitia boundary (MAB).</td>
</tr>
</tbody>
</table>

TABLE II: Full Description of Labeling of Fig. 5

<table>
<thead>
<tr>
<th>Label</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROI 1</td>
<td>N1 North; the midpoint of the Adventitia boundary</td>
</tr>
<tr>
<td></td>
<td>S1 South; the midpoint of the Media boundary</td>
</tr>
<tr>
<td></td>
<td>E1 East; the rightmost tip</td>
</tr>
<tr>
<td></td>
<td>W1 West; the leftmost tip</td>
</tr>
<tr>
<td></td>
<td>NE2 NorthEast; the path described by the arc on the adventitia boundary between N1 and E1</td>
</tr>
<tr>
<td></td>
<td>SE1 SouthEast; the path described by the arc on the media boundary between S1 and E1</td>
</tr>
<tr>
<td>ROI 2</td>
<td>N2 North; the midpoint of the media boundary</td>
</tr>
<tr>
<td></td>
<td>S2 South; the midpoint of the intima boundary</td>
</tr>
<tr>
<td></td>
<td>E2 East; the rightmost tip</td>
</tr>
<tr>
<td></td>
<td>W2 West; the leftmost tip</td>
</tr>
<tr>
<td></td>
<td>NE3 NorthEast; the path described by the arc on the media boundary between N2 and E2</td>
</tr>
<tr>
<td></td>
<td>SW2 SouthWest; the path described by the arc on the intima boundary between S2 and W2</td>
</tr>
<tr>
<td></td>
<td>SE2 SouthEast; the path described by the arc on the intima boundary between S2 and E2</td>
</tr>
<tr>
<td>ROI 3</td>
<td>N3 North; the midpoint of the intima boundary</td>
</tr>
<tr>
<td></td>
<td>S3 South; the midpoint of the media boundary</td>
</tr>
<tr>
<td></td>
<td>E3 East; the rightmost tip</td>
</tr>
<tr>
<td></td>
<td>W3 West; the leftmost tip</td>
</tr>
<tr>
<td></td>
<td>NW4 NorthWest; the path described by the arc on the intima boundary between N3 and W3</td>
</tr>
<tr>
<td></td>
<td>NE4 NorthEast; the path described by the arc on the intima boundary between N3 and E3</td>
</tr>
<tr>
<td></td>
<td>SW4 SouthWest; the path described by the arc on the intima boundary between S3 and W3</td>
</tr>
<tr>
<td></td>
<td>SE4 SouthEast; the path described by the arc on the intima boundary between S3 and E3</td>
</tr>
<tr>
<td>ROI 4</td>
<td>N4 North; the midpoint of the media boundary</td>
</tr>
<tr>
<td></td>
<td>S4 South; the midpoint of the adventitia boundary</td>
</tr>
<tr>
<td></td>
<td>E4 East; the rightmost tip</td>
</tr>
<tr>
<td></td>
<td>W4 West; the leftmost tip</td>
</tr>
<tr>
<td></td>
<td>NW5 NorthWest; the path described by the arc on the media boundary between N4 and W4</td>
</tr>
<tr>
<td></td>
<td>NE5 NorthEast; the path described by the arc on the media boundary between N4 and E4</td>
</tr>
<tr>
<td></td>
<td>SW5 SouthWest; the path described by the arc on the adventitia boundary between S4 and W4</td>
</tr>
<tr>
<td></td>
<td>SE5 SouthEast; the path described by the arc on the adventitia boundary between S4 and E4</td>
</tr>
</tbody>
</table>

III. DEVELOPING THE CPS-LDM EQUATIONS

The CPS model describes the geometric shape of the carotid arteries by two set of landmarks: The Fixed Landmarks (FLs) and the Movable Landmarks (MLs). The Total Landmarks (TLs) which completely describe the shape of the carotid arteries is given by equation (1).

\[ TLs = FLs + MLs \]  

A. Fixed landmarks (FLs) equation

Fig. 6 shows the FLs on the artery, there carotid are four FLs for each ROI which are the North (N), the South (S), the East (E) and the West (W). For example, ROI 1 has points on N1, S1, E1, and W1. These FLs are the red points marked on the ROIs of Fig. 6.

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Fig. 6. Positions of the FLs on the carotid artery

Let the number of ROI(s) desired to be described be represented by \( U \), and then position of the described ROI(s) be represented by index \( M \), then the number of FLs on any desired ROI is given by equation (2).

\[
FLs = 4U_M
\]

Equation (2) governs the annotating of fixed landmarks on the carotid artery image.

B. Rules followed in selecting the index \( M \)

1) When the positions of the ROIs to be described follow each other in sequence, then a hyphen (-) is used in between the numbers to separate them e.g.
   - For \( M = 1-2 \); this means that ROI 1 and ROI 2 are described.
   - For \( M = 1-3 \); this means that ROI 1, ROI 2 and ROI 3 are described.
   - For \( M = 1-4 \); this means that ROI 1, ROI 2, ROI 3 and ROI 4 are described.
   - For \( M = 2-4 \); this means that ROI 2, ROI 3 and ROI 4 are described.
   - For \( M = 3-4 \); this means that ROI 3 and ROI 4 are described.

2) When the positions of the ROIs to be described do not follow each other sequentially, then a comma (,) is used in between the numbers to separate them e.g.
   - For \( M = 1, 3 \); this means that ROI 1 and ROI 3 are described.
   - For \( M = 1, 4 \); this means that ROI 1 and ROI 4 are described.
   - For \( M = 2, 4 \); this means that ROI 2 and ROI 4 are described.

3) A combination of the above two rules can also be used e.g.
   - For \( M = 1-2, 4 \); this means that ROI 1, ROI 2 and ROI 4 are described.
   - For \( M = 1, 3-4 \); this means that ROI 1, ROI 3 and ROI 4 are described.

The fixed landmarks (FLs) are not sufficient to fully capture the geometry of the carotid artery, hence the need for the movable landmarks (MLs).

C. Movable landmarks (MLs) equation

In developing the MLs equation, the symmetry of the carotid artery shape shown in Fig. 6 is utilized. Symmetry equations for each ROI given in equation (3-18) were developed.

For ROI 1

\[
\begin{align*}
NW_1 &= NE_1 \\
SW_1 &= SE_1 \\
NW_1 + NE_1 &= WNE_1 \\
SW_1 + SE_1 &= WSE_1
\end{align*}
\]

For ROI 2

\[
\begin{align*}
NW_2 &= NE_2 \\
SW_2 &= SE_2 \\
NW_2 + NE_2 &= WNE_2 \\
SW_2 + SE_2 &= WSE_2
\end{align*}
\]

For ROI 3

\[
\begin{align*}
NW_3 &= NE_3 \\
SW_3 &= SE_3 \\
NW_3 + NE_3 &= WNE_3 \\
SW_3 + SE_3 &= WSE_3
\end{align*}
\]

For ROI 4

\[
\begin{align*}
NW_4 &= NE_4 \\
SW_4 &= SE_4 \\
NW_4 + NE_4 &= WNE_4 \\
SW_4 + SE_4 &= WSE_4
\end{align*}
\]

Let \( n(K_{CP}^{ROI}) \) notation represent the number of integer points that can be annotated on the ROI boundaries of the carotid artery where \( n(K) \) is the number of integer points on the ROIs, ROI (superscript) is the position of the ROI under consideration (ROI = 1, 2, 3, 4) and CP (subscript) is the cardinal point under consideration (CP = NW, NE, etc.)

The MLs equation is then given as:

\[
MLs = n(K_{NW}^{1}) + n(K_{NE}^{1}) + n(K_{SW}^{1}) + n(K_{NE}^{1}) + n(K_{SW}^{1}) + n(K_{SE}^{2}) + n(K_{SW}^{2}) + n(K_{NE}^{2}) + n(K_{NW}^{2}) + n(K_{SE}^{3}) + n(K_{SW}^{3}) + n(K_{NE}^{3}) + n(K_{NW}^{3}) + n(K_{SE}^{4}) + n(K_{SW}^{4}) + n(K_{NE}^{4}) + n(K_{NW}^{4}) + n(K_{SE}^{4})
\]

(19)
Applying the symmetry equations (3 to 18) into equation (19) yield equation (20)

\[
MLs = n(K_{WNE}^{1,4}) + n(K_{WSE}^{1,4}) + n(K_{WNE}^{2,3}) + n(K_{WSE}^{2,3})
\]

\[
+ n(K_{WNE}^{4,1}) + n(K_{WSE}^{4,1}) \quad (20)
\]

D. ROI “mirror-image” condition

Looking carefully at Fig. 6, it can be observed that the horizontal line of symmetry HS-HS divides the image such that ROI 4 is like a mirror image of ROI 1 and ROI 3 is a mirror image of ROI 2. This leads to the postulation of the “mirror-image” condition for simulated ultrasound carotid artery imaged in the transverse cross-sectional plane which is stated below.

If an ultrasound carotid artery is properly and carefully simulated in the transverse cross-sectional plane such that all the ROIs are captured, the horizontal line of symmetry HS-HS divides the image such that then the posterior region is like the mirror image of the anterior region. i.e. ROI 4 is like a mirror image of ROI 1 and ROI 3 is like a mirror image of ROI 2.

Applying the “mirror-image” condition, equation (20) is further simplified into equation (21)

\[
MLs = n(K_{WNE}^{1,4}) + n(K_{WSE}^{1,4}) + n(K_{WNE}^{2,3}) + n(K_{WSE}^{2,3}) \quad (21)
\]

Equation (21) can be written in a matrix-like form as shown in (22).

\[
MLs = \begin{bmatrix}
K_{WNE}^{1,4} & K_{WNE}^{2,3} \\
K_{WSE}^{1,4} & K_{WSE}^{2,3}
\end{bmatrix}
\quad (22)
\]

Equation (22) governs the landmarking of movable points on the simulated carotid artery image.

E. Total landmarks (TLs) equation

The total number of landmarks that can be annotated on the transverse section of any simulated carotid artery image to govern its complete shape characterization is given in equation (1) which is repeated below.

\[
(TLs) = (FLs) + (MLs)
\]

From equation (2) and equation (22), TLs becomes

\[
TLs = 4U_{M} + \begin{bmatrix}
K_{WNE}^{1,4} & K_{WNE}^{2,3} \\
K_{WSE}^{1,4} & K_{WSE}^{2,3}
\end{bmatrix}
\quad (23)
\]

We called equation (23) the Cardinal Point Symmetry Landmark Distribution Model (CPS-LDM) Equation and this equation completely characterize the geometric shape of any simulated B-mode ultrasound carotid artery imaged in the transverse cross-sectional plane.

IV. CONCLUSION

In this paper, a new Cardinal Point Symmetry Landmark Distribution Model (CPS-LDM) was developed. This model was shown to be able to segment sufficiently the ROIs of B-mode ultrasound carotid artery simulated in the transverse plane. This model was also shown to be generic enough and adaptable to varieties of B-mode ultrasound carotid arteries simulated under various scenarios.

APPENDIX

In appendix A and B, examples are shown on how to use CPS-LDM Model to landmark and segment simulated carotid artery B-mode ultrasound images in the transverse plane.

A. Appendix A: Example 1

Given the equation

\[
TLs = 4U_{M} + \begin{bmatrix}
10 & 8 \\
8 & 6
\end{bmatrix}
\quad (24)
\]

Where U = 4 and M = 1-4. Calculate the TLs required to fully annotate the carotid artery and show its Shape Space Pattern (SSP).

B. Calculations, Results and Discussion

Comparing the given equation (24) with the CPS-LDM equation in (23), it is observed that the first term in the right hand side of equation (24) determines the FLs while the second term of the equation determines the MLs. The numbers imputed into the matrix of the MLs equation is user dependent. To determine the FLs, equation (2) given as equation (25) here is used.

\[
FLs = 4U_{M}
\quad (25)
\]

FLs = 4 x 4 = 16 landmarks

(M = 1 - 4; ROIs 1, 2, 3 and 4 are required to be described)

Each ROIs will contain 4 FLs.

Fig. 7 shows how the FLs are annotated on the carotid artery.
respectively.

| TABLE III: LANDMARK COORDINATES FOR THE FLS |
|-----------------|-----------------|-----------------|
| Label | Nature of labels | X-Coordinates | Y-Coordinates |
| 1 | W1 | 44.667 | 38.667 |
| 2 | N1 | 91.333 | 16.667 |
| 3 | S1 | 140.000 | 36.667 |
| 4 | E1 | 90.667 | 24.667 |
| 5 | W2 | 53.333 | 51.333 |
| 6 | N2 | 91.333 | 34.667 |
| 7 | S2 | 131.333 | 52.000 |
| 8 | E2 | 90.667 | 42.667 |
| 9 | W3 | 42.000 | 155.333 |
| 10 | N3 | 91.333 | 180.667 |
| 11 | S3 | 90.667 | 170.667 |
| 12 | E3 | 141.333 | 155.333 |
| 13 | W4 | 38.667 | 173.333 |
| 14 | N4 | 91.333 | 189.333 |
| 15 | S4 | 90.667 | 197.333 |
| 16 | E4 | 140.667 | 178.667 |

The MLs equation is given as:

\[
MLs = \begin{bmatrix}
K^{1,4}_{WNE} & K^{2,3}_{WNE} \\
K^{1,4}_{WSE} & K^{2,3}_{WSE}
\end{bmatrix}
\]  

where

\[
n(K^{1,4}_{WNE}) = 10, \quad n(K^{1,4}_{WSE}) = 8, \quad n(K^{2,3}_{WNE}) = 8, \quad n(K^{2,3}_{WSE}) = 6
\]  

The superscript 1, 4 and subscript WNE of the term \(n(K^{1,4}_{WNE}) = 10\) implies that ROIs 1 and 4 is divided equally into NW and NE respectively. The integer 10 is chosen by the user and it can be changed at the user’s discretion. The underlying principle in the MLs equation is that the integer 10 is divided also into two equal integers. These equal integers in this case will be landmarked on the NW and NE of ROIs considered at that point, in this case, ROIs 1 and 4. This procedure applies to all other terms and integers in the MLs equation. This operation is captured mathematically in equations (28) and (29).

\[
MLs = \begin{bmatrix}
ROI 1, 4 : [WNE] = 10 & ROI 2, 3 : [WNE] = 8 \\
ROI 1, 4 : [WSE] = 8 & ROI 2, 3 : [WSE] = 6
\end{bmatrix}
\]  

C. Spacing constraints when annotating the FLs and MLs together on the carotid artery image

According to our model, the carotid artery image is completely characterized by its number of FLs and MLs. Though the spacing between these points are intuitively...
determined by the user, the following spacing constraints are followed when annotating these landmarks on the carotid artery image.

1) The distance between any consecutive ML annotated along the path of the cardinal point of any chosen ROI should be equally spaced. For example from figure 9 in example 1, the spacing of the MLs in ROI 1, 4: along the NW cardinal point is constrained by the condition

\[ |ML_1 - ML_2| = |ML_2 - ML_3| = |ML_3 - ML_4| = \ldots = h_1 \]

The spacing of the MLs in ROI 1, 4: along the SW cardinal point is constrained by the condition

\[ |ML_1 - ML_2| = |ML_2 - ML_3| = |ML_3 - ML_4| = \ldots = h_2 \]

where \( h_1 \) and \( h_2 \) are any real number and \( h_1 \) and \( h_2 \) may be equal or not.

2) The distance between a FL and a consecutive ML either in the forward or backward direction along the path of the cardinal point of any chosen ROI should be equal to the distance between the consecutive MLs along that same ROI path. For example from figure 9 in example 1, the spacing of the FLs and MLs in ROI 1, 4: along the NW cardinal point is constrained by the condition

\[ |FL_1 - ML_1| = |ML_1 - ML_2| = |ML_2 - ML_3| = \ldots = h_1 \]

The spacing of the MLs in ROI 1, 4: along the NW cardinal point is constrained by the condition

\[ |FL_1 - ML_1| = |ML_1 - ML_2| = \ldots = |ML_3 - ML_4| = h_2 \quad \text{and so on}, \quad \text{where } h_1 \text{ and } h_2 \text{ are any Real number and } h_1 \text{ and } h_2 \text{ may be equal or not.} \]

3) Based on the above two conditions, all consecutive landmarks (either FLs and MLs) along the path of the cardinal point of any chosen ROI should be equally spaced so that consistency of annotation can be maintained on any set of numbers of images.

There are situations when the number chosen by the user for the MLs are not even integers, this poses a problem for equal integer divisibility. Example 2 in appendix B addressed how such challenge was overcome.

D. Appendix B: Example 2

Given the equation

\[ TLs = 4U_M + \begin{bmatrix} 9 & 7 \\ 8 & 5 \end{bmatrix} \] (31)

Where \( U = 4 \) and \( M = 1-4 \). Calculate the TLs required to fully annotate the carotid artery and show its Shape Space Pattern (SSP).

E. Calculations and Results

Comparing the given equation (31) with the CPS-LDM equation given in (23), the FLs equation is given as equation (32) here

\[ (FLs) = 4U_M \]

\[ FLs = 4 \times 4 = 16 \text{ landmarks} \]

\( (M = 1 - 4; \text{ ROIs 1, 2, 3 and 4 are required to be described).} \)

Each ROIs will contain 4 FLs.

Also comparing the given equation (31) with the CPS-LDM equation given in (23), the MLs equation given as equation (33) here is:

\[ MLs = \begin{bmatrix} K_{WNE}^{1.4} & K_{WNE}^{2.3} \\ K_{WSE}^{1.4} & K_{WSE}^{2.3} \end{bmatrix} \] (33)

where:

\[ n(K_{WNE}^{1.4}) = 9, \quad n(K_{WNE}^{2.3}) = 8, \quad n(K_{WSE}^{2.3}) = 7, \]

\[ n(K_{WSE}^{2.3}) = 5 \] (34)

then

\[ MLs = \begin{bmatrix} ROI 1, 4 : [WNE] = 9 & ROI 2, 3 : [WNE] = 7 \\ ROI 1, 4 : [WSE] = 8 & ROI 2, 3 : [WSE] = 5 \end{bmatrix} \] (35)

For \( n(K_{WNE}^{1.4}) = 9 \), symmetry equation demands that \( NE = 4.5 \), and \( NW = 4.5 \). But the number of points must be an integer, hence the symmetry equations will be modified to make one side greater than the other side by one integer. For our example above, the following choices can be made

(1) \( NE = 4 \) and \( NW = 5 \); or

(2) \( NE = 5 \) and \( NW = 4 \). (36)

Once any of the choice is chosen, it must be consistently maintained for all set of the carotid artery images. Going for the choice of (36), the MLs equation becomes

\[ MLs = \begin{bmatrix} ROI 1, 4 : NW = 5 & ROI 2, 3 : NW = 3 \\ ROI 1, 4 : NE = 4 & ROI 2, 3 : NE = 4 \\ ROI 1, 4 : SW = 4 & ROI 2, 3 : SW = 3 \\ ROI 1, 4 : SE = 4 & ROI 2, 3 : SE = 2 \end{bmatrix} \] (38)

\[ MLs = (2x5) + (2x4) + (2x4) + (2x3) + (2x4) + (2x3) + (2x2) \]

\[ MLs = 10 + 8 + 8 + 6 + 8 + 6 + 4 \]

\[ MLs = 58 \text{ Landmarks} \]

The total landmarks (TLs) that fully describe the geometry of the carotid artery based on equation (31) and given as (39) here is calculated thus

\[ (TLs) = (FLs) + (MLs) \] (39)

\[ TLs = 16 + 58 \]

\[ TLs = 74 \text{ Landmarks}. \]

Figure 11 shows the landmark distribution of both the
FLs and MLs on the simulated carotid artery image. Figure 12 shows the complete SSP of the carotid artery.

![Image](image1.png)

**Fig. 11.** Landmark distribution for both the FLs and MLs on the carotid artery image

![Image](image2.png)

**Fig. 12.** The complete SSP of the landmarked carotid artery image of Fig.

**REFERENCES**


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