Individual Universes of the Multiverse

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Abstract—Using exploded numbers we consider the exploded three-dimensional space as a mathematical model of the Multiverse. Our universe is only one among the infinite number three-dimensional individual universes of the Multiverse. We introduce the concept of box – phenomenon of objects outside our universe. Applying a shift coordinate transformation, we investigate the shifted box – phenomenon and show certain parts of objects in the Multiverse selected from in different individual universes. Among others we show that the same part of a given object yields different views depending on the individual universes. Moreover, we give a measure of distance concerning the Multiverse. Finally, we mention the super – distance of different individual universes.

Index Terms—Exploded and Compressed Numbers; Individual Universes; Box – Phenomenon.

I. INTRODUCTION

We visualise our universe as the familiar three dimensional Euclidean space

\[ \mathbb{R}^3 = \{(x, y, z) | (-\infty < x < \infty, -\infty < y < \infty, -\infty < z < \infty)\} \]

with its well-known apparatus, among others

- the ordered field \((\mathbb{R}, <, +, \cdot)\) of real numbers,
- the vector algebra of \(\mathbb{R}^3\), the multiplication \(c \cdot P = (cx, cy, cz), c \in \mathbb{R}\), the addition \(P_1 + P_2 = (x_1 + x_2, y_1 + y_2, z_1 + z_2)\), the inner product \(P_1 \cdot P_2\) the norm \(\|P\| = \sqrt{x^2 + y^2 + z^2}\), and distance \(d(P_1, P_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}\).

The apparatus of exploded and compressed numbers is described in [1] (See Section II.). Here we collect the main points:

- For any \(x \in \mathbb{R}\) its exploded is denoted by \(\bar{x}\). The set of exploded numbers is denoted by \(\overline{\mathbb{R}}\). The set \(\overline{\mathbb{R}}\) is a proper subset of \(\mathbb{R}\). For any \(x \in [-1,1]\)

  \[ \bar{x} = \tanh^{-1} x = \frac{1}{2} \ln \frac{1 + x}{1 - x} \] (1)

- The set \(\overline{\mathbb{R}}\) has an algebraic structure with the super − operations

  \(\bar{x} \oplus \bar{y} = \bar{x} + \bar{y} \quad ; x, y \in \overline{\mathbb{R}}\) (2)

and

\(\bar{x} \odot \bar{y} = \bar{x} \cdot \bar{y} \quad ; x, y \in \overline{\mathbb{R}}\) (3)

called super − addition and super − multiplication, respectively.

- For any pair \(\bar{x}, \bar{y} \in \overline{\mathbb{R}}\) we say that \(\bar{x} = \bar{y}\) if and only if \(x = y\) and \(\bar{x} \leq \bar{y}\) if and only if \(x \leq y\). (Monotony of explosion.) Hence, we have that \((\overline{\mathbb{R}}, <, \oplus, \odot)\) is an ordered field which is isomorphic with \((\mathbb{R}, <, +, \cdot, \cdot)\). For any \(x \in \mathbb{R}\) then (0.2) with (0.1) yields that \(\bar{x} \odot (-\bar{x}) = \bar{0} = 0\). Moreover, if \(x \in [-1,1]\) then we have \((-\bar{x}) = -\bar{x}\). So, we may denote the super − additive inverse of \(\bar{x}\) by \(-\bar{x}\) for any \(x \in \overline{\mathbb{R}}\). (If \(x \in [-1,1]\) then the additive inverse and super − additive inverse of \(\bar{x}\) are equal. But if \(x \in \mathbb{R} \setminus [-1,1]\) then \(\bar{x}\) is not a real number and its additive inverse does not exist.)

- For any \(u \in \overline{\mathbb{R}}\) the real number \(u\) is called the compressed of \(u\) defined by the (first) inversion formula,

  \[ \widehat{u} = u \quad , u \in \overline{\mathbb{R}}. \] (4)

Hence, for any \(u \in \overline{\mathbb{R}}\) we have

\[ u = \tanh u \left(\frac{u}{e^{u} + e^{-u}}\right) \quad , u \in \overline{\mathbb{R}}. \] (5)

- Using the inversion formula (4) we give the super − operations new forms (see (2) and (3)).

\[ u \oplus v = \widehat{u} + \widehat{v} \quad ; u, v \in \overline{\mathbb{R}} \] (6)

and

\[ u \odot v = \widehat{u} \cdot \widehat{v} \quad ; u, v \in \overline{\mathbb{R}} \] (7)

Moreover,

\[ (\bar{u}) = -\bar{u} \quad , u \in \overline{\mathbb{R}}. \] (8)

- The inversion formula (4) yields the (second) inversion formula

  \[ \bar{x} = x \quad , x \in \overline{\mathbb{R}}. \] (9)

The exploded numbers \((-1)\) and \(1\) are not real numbers. \((\overline{1})\) is the greatest exploded number which is smaller than each number and \(\overline{1}\) is the smallest exploded number which is greater than each number. \((-1)\) and \(\overline{1}\) are called negative and positive discriminators, respectively. Clearly, the exploded number \(u(\in \overline{\mathbb{R}})\) is a real number that is \(-\infty < u < \infty\) if and only if \(-1 < u < 1\).

Considering a set of points of our universe

\[ \mathcal{S} = \{(x, y, z) | x, y, z \in \mathbb{R}\} \]

the set

\[ \overline{\mathcal{S}} = \{\bar{x}, \bar{y}, \bar{z}) | x, y, z \in \overline{\mathbb{R}}\} \]

is called the exploded of \(\mathcal{S}\).
In this paper the model of the Multiverse is the exploded set of our universe \( \mathbb{R}^3 \). The Multiverse is denoted by \( \mathbb{R}^3 \).\]

Considering a set of points of the Multiverse
\[ \mathfrak{M} = \{ \mathcal{P} = (u, v, w) \mid u, v, w \in \mathbb{R} \} \]
the set
\[ \mathfrak{M} = \{ \mathcal{P} = (u, v, w) \mid u, v, w \in \mathbb{R} \} \]
is called the compressed of \( \mathfrak{M} \).\]

Clearly, the compressed of our universe is the open cube:
\[ \mathbb{R}^3 = \{ \mathcal{P} = (x, y, z) \mid -1 < x < 1, -1 < y < 1, -1 < z < 1 \} \]

\[ \text{Fig. 1. Some compressed lines in the compressed universe} \]

II. THE BOX–PHENOMENON

If we explode the compressed universe \( \mathbb{R}^3 \) we have our universe
\[ \mathbb{R}^3 = \{ \mathcal{P} = (u, v, w) \mid -1 < u < 1, -1 < v < 1, -1 < w < 1 \} \]
again. Of course, \( \mathbb{R}^3 \) is a subset of the Multiverse \( \mathbb{R}^3 \). We can consider it as an open „big” box inside the Multiverse. If one of the coordinates of point \( \mathcal{P} \in \mathbb{R}^3 \) is greater than or equal to \( 1 \) or less than or equal to \( -1 \) then \( \mathcal{P} \) is outside our universe. So, there are a lot of sets in the Multiverse which – partly or in full - are not seen in our universe. For example, the borders of our universe are such subsets of the Multiverse. The compressed of its borders are seen in Fig. 1. Namely, they are the six border – surfaces of the open cube. Taking a subset \( \mathfrak{M} \) of the Multiverse the section
\[ \mathfrak{M}_{\text{box}} = \mathfrak{M} \cap \mathbb{R}^3 \]
is called the box – phenomenon of \( \mathfrak{M} \). Clearly,
\[ \mathbb{R}^3_{\text{box}} = \mathbb{R}^3 = \mathbb{R}^3_{\text{box}} \]

A. Example 1

Let us consider the super – plane \( \mathcal{S} \) described by:
\[ w = u \oplus v, \ u, v \in \mathbb{R} \]
(11)

In the case of \( \mathcal{S}_{\text{box}} \) we have that \( -1 < u < 1, -1 < v < 1 \) and
\[ -1 < u \oplus v < 1. \]
By (6) we can write
\[ -1 < u \oplus v < 1. \]

Considering the monotonity of explosion, \(-1 < u < 1, -1 < v < 1\) and
\[ -1 < u + v < 1 \]
are obtained. Hence, (5) gives that \(-1 < \tanh u + \tanh v < 1\).
Moreover, if for coordinate \( w \) (see (11)) we use the inversion formula (4) by (5) we get that \( \mathcal{S}_{\text{box}} \) is described by:
\[ w = \tanh^{-1}(\tanh u + \tanh v), \ u, v \in \mathbb{R}. \]
(12)

\[ \text{Fig. 2. The box-phenomenon of this super – plane is not a plane} \]

B. Example 2

Let us consider plane \( \mathbb{S} \), represented by:
\[ z = x + y, \ x, y \in \mathbb{R} \]
(13)
We state that the exploded of plane \( S \) is super – plane \( \mathcal{M} \) given by (11), that is
\[
\tilde{S} = \mathcal{M}
\]
For \( u, v, w \in \mathbb{R} \) we introduce the nominations
\[
u = \tilde{x}, \; \tilde{v} = \tilde{y} \text{ and } w = \tilde{z}
\]
and using (2) and (13) we can write:
\[
w = \tilde{z} = \tilde{x} + \tilde{y} = \tilde{x} \odot \tilde{y} = u \odot v.
\]
Hence, (11) proves our statement.

Now, we consider the super - surface \( \tilde{S} \) given by:
\[
w = (u \odot v) \odot (1 \odot (u \odot v)), \; u, v \in \mathbb{R} \text{ such that } u \cdot v \neq -1
\]
where the super – division is defined by
\[
\tilde{x} \odot \tilde{y} = \left( \frac{\tilde{x} \cdotp \tilde{y}}{\tilde{y}} \right), \; x, y \neq 0 \in \mathbb{R}
\]

Starting from (15) we remark that easy to see that \( 1 \odot (u \odot v) \neq 0 \). Computing with the super - operations under (6), (7) and (16) we write
\[
w = (u \odot v) \odot (1 \odot (u \odot v)) = \frac{u + v}{1 + (u \cdot v)} \odot (1 + (1 + u \cdot v)) = \frac{u + v}{1 + (u \cdotp v)}
\]
Considering the box – phenomenon \( \mathcal{B}_{\text{box}} \) we assume that \( u \) and \( v \) are real numbers. Applying (5) we have
\[
w = \left( \frac{\tanh u + \tanh v}{1 + (\tanh u \cdotp \tanh v)} \right) = \tanh(u + v).
\]
As \(-1 < \tanh(u + v) < 1\) we may apply (0.1), so
\[
w = u + v, \; u, v \in \mathbb{R}.
\]
Hence, by (13) we can see that box – phenomenon of the surface \( \tilde{S} \) is just plane \( S \), that is
\[
\tilde{S} = \tilde{S}_{\text{box}}.
\]

C. Example 3

we can see that the box – phenomenon of the closed super – ball \( \mathcal{B} \) demonstrated by:
\[
(u \odot u) \odot (v \odot v) \odot (w \odot w) = 1, \; u, v, w \in \mathbb{R}, \quad (17)
\]
is “almost” the super – ball itself. Really the points of \( \mathcal{B} \), except the points
\[
\mathcal{P}_{\text{before}} = (1, 0, 0), \; \mathcal{P}_{\text{right}} = (0, 1, 0), \; \mathcal{P}_{\text{above}} = (0, 0, 1),\quad \mathcal{P}_{\text{below}} = (0, -1, 0),
\]
are the points of \( \mathcal{B}_{\text{box}}, \) too. To prove this observation we seek the equation which describes the \( \mathcal{B}_{\text{box}} \). Computing with the super - operations under (7) and (6) by (17)
\[
(u - u) \odot (v - v) \odot (w - w) = 1
\]
is obtained. Having the inversion formula (9) we get
\[
(u^2 + (v)^2 + (w)^2 = 1
\]
Looking at the box – phenomenon \( \mathcal{B}_{\text{box}} \) we assume that \( u, v \) and \( w \) are real numbers. Applying (5) we have
\[
(tanh u)^2 + (tanh v)^2 + (tanh w)^2 = 1
\]
which proves the observation mentioned above.

Considering ball \( \mathcal{B} \) given by:
\[
x^2 + y^2 + z^2 = 1, \; x, y, z \in \mathbb{R}
\]
we state that \( \tilde{\mathcal{B}} = \mathcal{B} \). To prove this statement we must find a necessary and sufficient connection among \( \tilde{x}, \; \tilde{y}, \; \tilde{z} \) by the condition (19). Exploding the both side of under (19) and using (2) and (3)
\[
(\tilde{x} \odot \tilde{x}) \odot (\tilde{y} \odot \tilde{y}) \odot (\tilde{z} \odot \tilde{z}) = 1
\]
is obtained. Using the nominations under (14) we have the (17). So our statement is proved. ■

D. Example 4

Let \( u \) and \( v \) be real numbers and \( m \) is an arbitrary positive exploded number. The set
\[
\mathcal{D}(m) = \{(u, v, w) \in \mathbb{R}^3 | ||u| \odot ||v| \leq 1 \ \text{and} \ w \}
\]
where the super – subtraction is defined by
\[
\tilde{x} \odot \tilde{y} = x - y, \; x, y \in \mathbb{R}
\]
is called super – pyramid, because its compressed is the pyramid
\[
\mathcal{D}(m) = \{(x, y, z) \in \mathbb{R}^3 | ||x| + ||y| \leq \text{tanh} 1 \ \text{and} \ z
\]
\[
= m \left( 1 - \frac{||x|}{\text{tanh} 1} - \frac{||y|}{\text{tanh} 1} \right)
\]
If \( m \) is a real number, then \( w \) is also a real number. So, the super – pyramid is bounded in our universe \( \mathbb{R}^3 \). For example in the case \( m = 2 \) we have Fig. 5.

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III. Universes Different from Our Universe

Considering a fixed point \( O_0 = (u_0, v_0, w_0) \) of the Multiverse \( \mathbb{R}^3 \) the super – shift transformation was introduced in [3] as follows:

\[
\begin{align*}
\xi &= u \quad \bigoplus \quad u_0 \\
\eta &= v \quad \bigoplus \quad v_0 \\
\zeta &= w \quad \bigoplus \quad w_0 \\
(u, v, w) &\in \mathbb{R}^3
\end{align*}
\]

The super – shift transformation instead of the exploded Descartes - coordinate system „\( u, v, w \)“ gives another system „\( \xi, \eta, \zeta \)” which has the origo \( O_0 \). The original origo \( \mathcal{O} = (0,0,0) \) which is the origo of our universe, has new coordinates, namely

\[
\mathcal{O} = (u = 0, v = 0, w = 0) = (\xi = -u_0, \eta = -v_0, \zeta = -w_0).
\]

(Here we remind the Reader, that the super – additive inverse was mentioned in the Introduction) To prevent the misunderstanding regarding the fixed point of the Multiverse we will give the coordinates in both systems. For example,

\[
O_0 = (u = u_0, v = v_0, w = w_0) = (\xi = 0, \eta = 0, \zeta = 0)
\]

We choose the following subset of the Multiverse:

\[
\mathcal{W}_{O_0} = \left\{ (u, v, w) \in \mathbb{R}^3 \mid -1 < u < 1, -1 < v < 1, -1 < w < 1 \right\}
\]

\[
O_0 = (u_0, v_0, w_0) \in \mathbb{R}^3
\]

If \( O_0 = \mathcal{O} \) then \( \mathcal{W}_{O_0} \) is just the our universe, given by (10). Otherwise our universe differs from \( \mathcal{W}_{O_0} \). Using the new coordinates given by the super - shift transformation

\[
\mathcal{W}_{O_0} = \left\{ (\xi, \eta, \zeta) \in \mathbb{R}^3 \mid -1 < \xi < 1, -1 < \eta < 1, -1 < \zeta < 1 \right\}
\]

is obtained, which shows that \( \mathcal{W}_{O_0} \) is a three – dimensional universe of the Multiverse. We can say that universe \( \mathcal{W}_{O_0} \) is an individual universe parallel to our universe.

Having a subset \( \mathcal{W} \) of the Multiverse the section

\[
\mathcal{W}_{\text{shifted-\text{box}}} = \mathcal{W} \cap \mathcal{W}_{O_0}
\]

is called the shifted box – phenomenon of \( \mathcal{W} \), concerning the universe \( \mathcal{W}_{O_0} \). Clearly,

\[
\mathbb{R}^3_{\text{shifted-\text{box}}} = \mathcal{W}_{O_0} \text{ and } \bigcup_{\forall O_0 \in \mathbb{R}^3} \mathcal{W}_{O_0} = \mathbb{R}^3
\]

The latter formula shows that by the shifted box – phenomena, like a moving three – dimensional universum – wide camera, we are able to discover the Multiverse in part.

A. Example 1

Fig. 2 makes it perceptible that the box – phenomenon \( \mathcal{W}_{\text{box}} \) described by (12) is boundless in our universe. More exactly, its borders (exploded border – passages of the regular hexagon, seen in Fig. 3), are already invisible in our universe.

Fig. 7 shows two cubes:
Below is the compressed of our universe $\mathbb{R}^3$ (see Fig. 1) and above is the cube

$$P = (x, y, z) \begin{cases} 0 < x < 2 \\ 0 < y < 2 \\ 0 < z < 2 \end{cases}$$

which is the compressed of the parallel universe

$$\mathcal{W}_0 = \{(u, v, w) \in \mathbb{R}^3 \mid 0 < u < 2, 0 < v < 2, 0 < w < 2\}.$$  

(See (14) and (20) with $u_0 = v_0 = w_0 = 1$.) We would like to see the shifted box – phenomenon $\mathcal{G}_{\text{shifted box}}$ of the super – plane $\mathcal{S}$ given by (11), in the universe the $\mathcal{W}_0$.

Applying the super – shift transformation

$$\begin{align*} \xi &= u \odot 1 \\ \eta &= v \odot 1 \\ \zeta &= w \odot 1 \end{align*}$$

(11) yields

$$\xi \odot 1 = (\xi \odot 1) \odot (\eta \odot 1).$$

Hence,

$$\zeta = \xi \odot \eta \odot 1.$$  

Applying (6) and using the inversion formula (9) we get

$$\zeta = \xi + \eta + 1.$$  

By (21) the super – shift transformation shows that $\xi, \eta, \zeta$ are between the positive and negative discriminators. So, they are real numbers. By (5) we have

$$\tanh \zeta = \tanh \xi + \tanh \eta + 1.$$  

Hence, the equation of $\mathcal{G}_{\text{shifted box}}$ is

$$\zeta = \tanh^{-1}(\tanh \xi + \tanh \eta + 1), \text{ where } (-2 <) \tanh \xi + \tanh \eta < 0.$$  

(22)

**Remark:** Comparing Fig. 2 and Fig. 8 we are not surprised that in the different universes the super – plane $\mathcal{S}$ has different views. Among others $\mathcal{O} = (0, 0, 0) \in \mathbb{R}^3$ but $\mathcal{O} \notin \mathcal{W}_0$ and $\mathcal{O}_0 = (1, 1, 1) \in \mathcal{W}_0$, but $\mathcal{O}_0 \notin \mathbb{R}^3$.

On the other hand, we can see that

$$\mathbb{R}^3 \cap \mathcal{W}_0 = \left\{ P = (x, y, z) \mid \begin{cases} 0 < x < 1 \\ 0 < y < 1 \\ 0 < z < 1 \end{cases} \right\}.$$  

So, the two universes $\mathbb{R}^3$ and $\mathcal{W}_0$ have a common part

$$\mathbb{R}^3 \cap \mathcal{W}_0 = \left\{ P = (u, v, w) \mid \begin{cases} 0 < u < \infty \\ 0 < v < \infty \\ 0 < w < \infty \end{cases} \right\}.$$  

Comparing Fig. 2 and Fig. 8 and observing the common part $\mathbb{R}^3 \cap \mathcal{W}_0$ we can see that the same part of super – plane $\mathcal{S}$ has different views. This fact is not surprising because the points of view $\mathcal{O}$ and $\mathcal{O}_0$ are different.

By the segment $\mathbb{R}^3 \cap \mathcal{W}_0 \cap \mathcal{S}$ we establish that the views of the same set depends not only on the individual universes but the points of view, too.

**B. Example 2**

We will investigate a line of our universe $\mathbb{R}^3$ which continues in another individual universe. Let us consider a simple super- plane $\mathcal{S} = \mathbb{R}^3$ determined by:

$$u = v, u, v \in \mathbb{R}$$  

(23)

Of course, $\mathcal{S} = \mathbb{R}^3$. Having the super – surface $\mathcal{F}$ given by (15) we consider the super – curve

$$\mathcal{F} \cap \mathcal{S} = \left\{ (u, v, w) \in \mathbb{R}^3 \mid \text{w} = \text{w}(\text{u}, \text{v}, \text{w}) \right\}.$$  

(24)

Using the super – operations (6), (7) and (16) by (24):

$$w = \left(\frac{2u}{v} \right), \ u \in \mathbb{R}^3$$  

(25)
is obtained. Looking for the \((\mathfrak{H} \cap \mathfrak{S}^n)_{\text{box}}\) we assume that \(u\) and \(v\) are real numbers, (25) with (5) and (1) yields:
\[
w = \left(\frac{2 \tanh u}{1 + \left(\tanh u\right)^2}\right) = \tanh 2u = 2u, \quad u \in \mathbb{R}.
\]

Using the three-dimensional \(\omega, \nu, \nu\) coordinate system of our universe (where the coordinates are real numbers), we introduce the \(\omega - \text{axis}\) as the section of the planes \(w = 0\) and \(u = v\), \(u, v \in \mathbb{R}\).

It is easy to see that \(\omega = u \cdot \sqrt{2}\), in the two-dimensional rectangular Descartes \(\omega, w\) system the \((\mathfrak{H} \cap \mathfrak{S}^n)_{\text{box}}\) has the equation:
\[
w = \sqrt{2} \cdot \omega, \quad -\infty < \omega < \infty
\]
So, \((\mathfrak{H} \cap \mathfrak{S}^n)_{\text{box}}\) forms a line illustrated by Fig. 9.

Next we seek the continuation of this line in the individual universe:
\[
\mathcal{W}_{\theta_0} = \left\{(u, v, w) \in \mathbb{R}^3 \mid \begin{array}{l}
-\frac{1}{2} < u < \frac{3}{2} \\
-\frac{1}{2} < v < \frac{3}{2} \\
-\frac{1}{2} < w < \frac{3}{2}
\end{array} \right\}
\]
\[
\sigma_0 = \left\{(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}) \in \mathbb{R}^3 \subset \mathbb{R}^2 \right\}
\]
(See (20) with \(u_0 = v_0 = w_0 = \frac{1}{2}\).)

We will use the super-shift transformation
\[
\begin{align*}
\xi &= u \odot \frac{1}{2} \\
\eta &= v \odot \frac{1}{2} \\
\zeta &= w \odot \frac{1}{2}
\end{align*}
\]
and casting a glance at (24), we compute by (2), (3), (6), (7), (16) and inversion formulas (4) and (9)
\[
w = (u \odot v) \odot \left(1 \odot (u \odot v)\right)
\]
\[
\left(\frac{\zeta + 1}{2}\right) = \left(\frac{\xi + \eta + 1}{\xi \cdot \eta + \frac{\xi + \eta}{2} + 5}\right)
\]
\[
\left(\frac{\zeta + 1}{2}\right) = \frac{\xi \cdot \eta + \frac{\xi + \eta}{2} + 5}{\xi \cdot \eta + \frac{\xi + \eta}{2} + 5}
\]
\[
\zeta = \frac{4 \cdot \xi \cdot \eta - 6 \cdot \left(\xi + \eta\right) - 3}{8 \cdot \xi \cdot \eta + 4 \cdot \left(\xi + \eta\right) + 10}
\]
is obtained. So, in place of (24) we get:
\[
\mathfrak{H} \cap \mathfrak{S}^n = \left\{(\xi, \eta, \zeta) \in \mathbb{R}^3 \mid \frac{4 \xi \cdot \eta - 6 \left(\xi + \eta\right) - 3}{8 \xi \cdot \eta + 4 \left(\xi + \eta\right) + 10} \right\}
\]
Looking for the \((\mathfrak{H} \cap \mathfrak{S}^n)_{\text{shifted box}}\) we assume that \(\xi\) and \(\eta\) are real numbers, (28) with (5) and (1) yields that:
\[
\zeta = -\tanh^{-1} \frac{4 \left(\tanh \zeta\right)^2 - 12 \tanh \xi - 3}{8 \left(\tanh \zeta\right)^2 + 8 \tanh \xi + 10}, \quad \zeta \in \mathbb{R}
\]
Using the three-dimensional \(\xi, \eta, \zeta\) coordinate system of universe \(\mathcal{W}_{\theta_0}\) (where the coordinates are real numbers), we introduce the \(\psi - \text{axis}\) as the section of the planes \(\zeta = 0\) and \(\xi = \eta\), \(\xi, \eta \in \mathbb{R}^3\).

It is easy to see that \(\psi = \xi \cdot \sqrt{2}\), in the two-dimensional rectangular Descartes \(\psi, \zeta\) system the \((\mathfrak{H} \cap \mathfrak{S}^n)_{\text{shifted box}}\) has the equation:
\[
\zeta = -\tanh^{-1} \frac{4 \left(\tanh \psi\right)^2 - 12 \tanh \psi - 3}{8 \left(\tanh \psi\right)^2 + 8 \tanh \psi + 10}, \quad \psi \in \mathbb{R}
\]
The full-sized \((\mathfrak{H} \cap \mathfrak{S}^n)_{\text{shifted box}}\) is illustrated by Fig. 10.
\[
\begin{align*}
\xi &= u \odot \left(\frac{1}{2}\right), \\
\eta &= v \odot \left(\frac{1}{2}\right), \\
\zeta &= w \odot \left(\frac{1}{2}\right), \\
(u, v, w) &\in \mathbb{R}^3
\end{align*}
\]
gives the coordinates of \( Q \) in the three-dimensional \( \xi, \eta, \zeta \) coordinate-system of universe \( \mathbb{W}_{00} \). Namely,
\[
Q = (u = 1, v = 1, w = 1) = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right).
\]
Similarly,
\[
O = (u = 0, v = 0, w = 0) = \left(-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right)
\]
and
\[
O_0 = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) = (0, 0, 0, 0)
\]
In Fig. 9 it is important to observe that the half-closed straight square \([O, Q]\) seems to be endless while in Fig 10 the closed square \([O, Q]\) is finite and becomes crooked. These facts seem to be antagonistic, but do not forget, that the forms depend on the points of view. Similarly, the Fig. 9 and Fig. 10 show the "local" measures. Each of them is valid in their own individual universe, only. (In Section IV we will give a measurement of distance concerning the Multiverse.)

C. Example 3

The super-ball \( \mathfrak{B} \) was already introduced in Section II Example 1. Equation (17) says that the point \( Q_{\infty} = (0, 0, 1) \) is situated on the border of the (closed) super-ball. Of course, \( Q_{\infty} \) is excluded from our universe \( \mathbb{R}^3 \). In the Fig.4 the spectacle of \( \mathfrak{B}_{box} \) suggests that the \( Q_{\infty} \) makes a peak – point. Next we investigate, whether this hypothesis is true or not. We will look at the \( \mathfrak{B}_{shifted \ box} \) in the individual universe:
\[
\mathbb{W}_{q_0} = \left\{(u, v, w) \in \mathbb{R}^3 \mid \begin{array}{l}
-1 < u < 1 \\
0 < w < 2
\end{array}, Q_0 = (0, 0, 1) \in \mathbb{R}^3 \right\} \tag{31}
\]
Using the shift – box transformation,
\[
\begin{align*}
\xi &= u \\
\eta &= v \\
\zeta &= w \odot 1
\end{align*}
\]
(17) yields:
\[
\xi \odot \eta \odot \zeta (\eta \odot \zeta) (\zeta \odot \zeta) = 0, \quad \xi, \eta, \zeta \in \mathbb{R} \tag{32}
\]
Looking at the \( \mathfrak{B}_{shifted \ box} \) we assume that \( \xi, \eta \) and \( \zeta \) are real numbers. Using (6) and (7) by (5), (32) yields:
\[
(tan \zeta)^2 + (tan \eta)^2 + (tan \xi)^2 + 2 \cdot tan \zeta = 0, \quad \xi, \eta, \zeta \in \mathbb{R} \tag{33}
\]
The point \( Q_{\infty} = (u = 0, v = 0, w = 1) = (\xi = 0, \eta = 0, \zeta = 0) \) satisfies the equation (33), that is \( Q_{\infty} \in \mathbb{W}_{q_0} \).

\[\mathfrak{B}_{shifted \ box}: \] Moreover, by (33) the computer gives the following Fig. 11:

\[\text{Fig. 11. Point } Q_{\infty} \text{ is situated on top of the picture}\]

The result of investigation that \( Q_{\infty} \) is not a peak – point. The inhabitants of the universe \( \mathbb{W}_{q_0} \) are able to smooth the super-ball on the border of our universe.

D. Example 4

We return to the Babel – tower \( \mathfrak{D} \left(\frac{3}{2}\right) \) introduced in Fig. 1.5.2. We are curious to see its continuation. First we will look at it in the individual universe \( \mathbb{W}_{q_0} \) (see (31)), again. First of all, we would like to see the break – through of the border of our universe. (Fig. 6 shows it approximately.)

As, the "invisible upper limit" of our universe \( \mathbb{R}^3 \) is situated on the super-plane determined by the equality \( w = 1 \), we will cut the body of the Babel – tower \( \mathfrak{D} \left(\frac{3}{2}\right) \) by this super-plane (See Fig. 1.5.2). So, the section is:
\[
\mathfrak{D} \left(\frac{3}{2}\right) = \left\{(u, v, 1) \in \mathbb{R}^3 \mid |u| |v| \leq 1 \right\} \tag{34}
\]
which forms a super-square (on the border of our universe) demonstrated by:
\[
\frac{3}{2} |u| |v| \odot (|u| |v| |1|) = \left(\frac{3}{2}\right) \tag{35}
\]
which has a simpler form:
\[
(|u| |v| |1|) = \left(\frac{3}{2}\right) \tag{35}
\]
We may assume that \( u \) and \( v \) are real numbers. So, by (16), (6), (5) and the inversion formula (9), (35) yields:
\[
|\tan u| + |\tan v| = \tan h \frac{1}{3} \tag{36}
\]
By this equation the computer gives Fig. 12.
Following this way first we will deal with the shifted – box phenomenon $\mathcal{D} \left( \frac{3}{2} \right)_{\text{shifted box}}$ in the individual universe $\mathcal{W}_{Q_0}$ (see (31)). Of course, the base of $\mathcal{D} \left( \frac{3}{2} \right)_{\text{shifted box}}$ is the super – square $\mathcal{D} \left( \frac{3}{2} \right)_{\text{section}}$ but what is the continuation? Using the super – shift transformation,

$$
\begin{align*}
\xi &= u \\
\eta &= v \\
\zeta &= w \text{ or } 1,
\end{align*}
$$

Now, with respect to (36) we have:

$$
\mathcal{D} \left( \frac{3}{2} \right)_{\text{shifted box}} = \left\{ (\xi, \eta, \zeta) \in \mathbb{R}^3 | |\xi| + |\eta| \leq \frac{1}{2} \right\} \text{ and } \zeta
$$

and computing with the exploded number applying the already used method:

$$
\mathcal{D} \left( \frac{3}{2} \right)_{\text{shifted box}} = \left\{ (\xi, \eta, \zeta) \in \mathbb{R}^3 | |\tan \xi| + |\tan \eta| \leq \frac{\tan 1}{3} \text{ and } \zeta \right\}
$$

is obtained. Moreover, the computer gives Fig. 13.

After the first step, $\mathcal{D} \left( \frac{3}{2} \right) = \mathcal{D} \left( \frac{3}{2} \right)_{\text{box}} \cup \mathcal{D} \left( \frac{3}{2} \right)_{\text{shifted box}}$, is obtained. So, we have the Babel – tower in two pieces (See Fig.6 and Fig.13).

Second, we would like to see $\mathcal{D} \left( \frac{3}{2} \right)$ all in one. The individual universe

$$
\mathcal{W}_{Q_0} = \left\{ (u, v, w) \in \mathbb{R}^3 \left\vert \begin{array}{c}
-1 < u < \frac{1}{2} \\
-1 < v < \frac{1}{2} \\
-\frac{1}{2} < w < \frac{3}{4}
\end{array} \right. \right\}
$$

$Q_0 = \left( 0,0,\left( \frac{3}{4} \right) \right) \in \mathbb{R}^3.$ (37)

and the super – shift transformation

$$
\begin{align*}
\xi &= u \\
\eta &= v \\
\zeta &= w \text{ or } 1
\end{align*}
$$

serve this purpose.

Applying (37) the Babel – tower $\mathcal{D} \left( \frac{3}{2} \right)$ has the following expression

$$
\mathcal{D} \left( \frac{3}{2} \right)_{\text{shifted box}} = \left\{ (\xi, \eta, \zeta) \in \mathbb{R}^3 | |\tan \xi| + |\tan \eta| \leq \frac{\tan 1}{3} \text{ and } \zeta \right\}
$$

is obtained. Moreover, the computer gives Fig. 14.
1.4 and 11.9), as in general. For example the traditional distance of the origo useful in our universe, but it is unusable for the Multiverse. The aim of, IV.

\[ \mathcal{O}(0) = (u = 0, v = 0, w = 0) \]

and the peak point of the Babel tower \( \mathfrak{D}(\frac{3}{2}) \) is:

\[ \mathcal{P} = (u = 0, v = 0, w = \frac{3}{2}) = (\xi = 0, \eta = 0, \zeta = \frac{3}{2}) \]

In Fig.14, in the depth, \( \zeta = \frac{3}{2} \) we find the base of the Babel tower which is situated in our universe, too. (See Fig. 6.). In the high part \( \zeta = \frac{2}{3} \) is the „Bible level”, which is situated in the individual universe \( \mathfrak{W}_0, \) too. (See Fig. 13.) The aim of Babel tower is written in the Scriptures (see The Book of Creation, 11.4 and 11.9), as shown by Fig. 12.

### IV. DISTANCE OF POINTS OF MULTIVERSE

The familiar Euclidean measure of distance, which is very useful in our universe, but is unusable for the Multiverse, in general. For example the traditional distance of the origo \( \mathcal{O}(0,0,0) \) and the point \( \mathcal{O}_0 = (0,0,1) \) does not exist. So, we need the concept of the super – distance. For reason we introduce the following concepts:

1. **The super absolute value of the exploded number** \( u \), denoted by \( \mid u \mid \), such that

\[
\mid u \mid = \begin{cases} u & \text{if } u > 0 \\ 0 & \text{if } u = 0 \\ -u & \text{if } u < 0 \end{cases}
\]

(the additive inverse element was already mentioned in the Introduction). Clearly, if \( u \) is a real number then \( \mid u \mid = \mid u \mid. \) Let \( \gamma, \gamma_1, \gamma_2 \) be exploded numbers and let \( \mathcal{P} = (u, v, w), \mathcal{P}_1 = (u_1, v_1, w_1), \mathcal{P}_2 = (u_2, v_2, w_2) \) ... and so on, be the points of Multiverse.

2. **\( \mathcal{P}_1 \oplus \mathcal{P}_2 = (u_1 \oplus u_2, v_1 \oplus v_2, w_1 \oplus w_2) \).** The properties,

\[
\begin{align*}
\mathcal{P}_1 \oplus \mathcal{P}_2 &= \mathcal{P}_2 \oplus \mathcal{P}_1 \\
(\mathcal{P}_1 \oplus \mathcal{P}_2) \oplus \mathcal{P}_3 &= \mathcal{P}_1 \oplus (\mathcal{P}_2 \oplus \mathcal{P}_3) \\
\mathcal{P} \oplus \mathcal{O} &= \mathcal{P} \\
\mathcal{P}(\mathcal{P} = \mathcal{O}) \rangle, where \ - \mathcal{P} = (-u, -v, -w)
\end{align*}
\]

are valid.

3. **\( \gamma \ominus \mathcal{P} = (\gamma \ominus u, \gamma \ominus v, \gamma \ominus w) \).** The properties,

\[
\begin{align*}
\gamma \ominus (\mathcal{P}_1 \oplus \mathcal{P}_2) &= (\gamma \ominus \mathcal{P}_1) \ominus (\gamma \ominus \mathcal{P}_2) \\
(\gamma \ominus \mathcal{P}_1) \ominus \mathcal{P}_2 &= (\gamma \ominus \mathcal{P}_1) \ominus \mathcal{P}_2 \\
(\gamma \ominus \mathcal{P}_1) \ominus \mathcal{P}_2 &= (\gamma \ominus \mathcal{P}_1) \ominus \mathcal{P}_2
\end{align*}
\]

are valid. We may observe that \( -\mathcal{P} = -\mathcal{P} \).

4. **\( \mathcal{P}_1 \ominus \mathcal{P}_2 = (u_1 \ominus u_2, v_1 \ominus v_2, w_1 \ominus w_2) \).** We have the familiar properties and the connection to the traditional inner product,

\[
\begin{align*}
\mathcal{P}_1 \ominus \mathcal{P}_2 = (\mathcal{P}_1 \ominus \mathcal{P}_2, (\mathcal{P}_1 \ominus \mathcal{P}_2), & (\mathcal{P}_1 \ominus \mathcal{P}_2) = \mathcal{P}_1 \ominus \mathcal{P}_2, (\mathcal{P}_1 \ominus \mathcal{P}_2) = \mathcal{P}_1 \ominus \mathcal{P}_2 \\
\mathcal{P}_1 \ominus \mathcal{P}_2 = (\mathcal{P}_1 \ominus \mathcal{P}_2, (\mathcal{P}_1 \ominus \mathcal{P}_2), & (\mathcal{P}_1 \ominus \mathcal{P}_2) = \mathcal{P}_1 \ominus \mathcal{P}_2)
\end{align*}
\]

Having that \( \mathcal{P}_1 \ominus \mathcal{P}_2 \) for the super – norm we give.

5. **\( \| \mathcal{P} \| = (\| \mathcal{P} \|) \).** We can prove the following properties,

\[
\begin{align*}
\| \mathcal{P} \| \geq 0 & \text{ and } \| \mathcal{P} \| = 0 \iff \mathcal{P} = \mathcal{O} \\
\| \gamma \ominus \mathcal{P} \| &= \| \gamma \ominus \mathcal{P} \| \\
\| \mathcal{P}_1 \ominus \mathcal{P}_2 \| &\leq \| \mathcal{P}_1 \| \ominus \mathcal{P}_2 \|, (\text{Cauchy - inequality}) \\
\| \mathcal{P}_1 \ominus \mathcal{P}_2 \| &\leq \| \mathcal{P}_1 \ominus \mathcal{P}_2 \|, (\text{Minkowsky - inequality})
\end{align*}
\]

For example, the proof of Minkowsky – inequality by (6), (9) and (2) is

\[
\| \mathcal{P}_1 \ominus \mathcal{P}_2 \| = \| \mathcal{P}_1 \| \ominus \mathcal{P}_2 \| = \| \mathcal{P}_1 \ominus \mathcal{P}_2 \| = \| \mathcal{P}_1 \ominus \mathcal{P}_2 \| = \| \mathcal{P}_1 \ominus \mathcal{P}_2 \|
\]

We can say that the Multiverse \( \mathbb{E}^3 \) is a normed (Euclidean) space. Finally, we are able to give the super – distance of the points in the Multiverse.

6. **\( d_{\mathbb{E}}(\mathcal{P}_1, \mathcal{P}_2) = (d(\mathcal{P}_1, \mathcal{P}_2)) \) (40)**

\( (\mathcal{P}_1 \text{ and } \mathcal{P}_2 \text{ are the points of our universe and their distance was already mentioned in the Introduction.}) \)

We can prove the following properties \( d_{\mathbb{E}}(\mathcal{P}_1, \mathcal{P}_2) \geq 0 \) and \( d_{\mathbb{E}}(\mathcal{P}_1, \mathcal{P}_2) = 0 \iff \mathcal{P}_1 = \mathcal{P}_2, \ d_{\mathbb{E}}(\mathcal{P}_1, \mathcal{P}_2) = d_{\mathbb{E}}(\mathcal{P}_2, \mathcal{P}_1) \) Moreover, for any points \( \mathcal{P}_1, \mathcal{P}_2 \text{ and } \mathcal{P}_3 \) of the Multiverse \( d_{\mathbb{E}}(\mathcal{P}_1, \mathcal{P}_2) \leq d_{\mathbb{E}}(\mathcal{P}_1, \mathcal{P}_3) + d_{\mathbb{E}}(\mathcal{P}_2, \mathcal{P}_3) \), (triangular – inequality) is valid. We prove the triangular – inequality, only. By (2),
is obtained.

Remark: The super – distance is not an extension of the distance used in our universe. For example we consider the joint points \( P_3 = \left( -\tanh^{-1} \frac{\sqrt{2}}{\sqrt{3}}, -\tanh^{-1} \frac{\sqrt{2}}{\sqrt{3}}, -\tanh^{-1} \frac{\sqrt{2}}{\sqrt{3}} \right) \) and \( P_2 = \left( \tanh^{-1} \frac{\sqrt{2}}{\sqrt{3}}, \tanh^{-1} \frac{\sqrt{2}}{\sqrt{3}}, \tanh^{-1} \frac{\sqrt{2}}{\sqrt{3}} \right) \) of the super – ball demonstrated by (17) and the super – line \( L_\omega = \{ (u,v,w) \in \mathbb{R}^3 | u = \xi, v = \eta, w = \zeta \} \).

Now we can see that \( d(P_2, P_3) = 2\sqrt{3}\tanh^{-1} \frac{\sqrt{2}}{\sqrt{3}} = 2.281 \) (see Fig. 4). On the other hand \( P_2 = \left( -\frac{\sqrt{2}}{\sqrt{3}}, -\frac{\sqrt{2}}{\sqrt{3}}, -\frac{\sqrt{2}}{\sqrt{3}} \right) \) and \( P_3 = \left( \frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}} \right) \), so by (3.8) \( d_{\mathbb{R}^3}(P_2, P_3) = 2 \notin \mathbb{R} \).

Remark: The super – distance spans over the different universes. For example, the point \( Q = (0,0, \frac{3}{2}) \in \mathbb{R}^3 \) and \( P = (u = 0, v = 0, w = \frac{3}{2}) \in W_Q \) are situated in different individual universes, their super – distance is \( d_{\mathbb{R}^3}(Q, P) = \tanh^{-1} \frac{3}{4} = 0.972950757 \).

Remark: The super-distance is invariant for the super – shift transformation \( \begin{align*}
\xi &= u \square u_0 \\
\eta &= v \square v_0 \\
\zeta &= w \square w_0
\end{align*} \), \( (u,v,w), (u_0,v_0,w_0) \in \mathbb{R}^3 \).

To prove this statement we consider the points \( P_3 = (u_1,v_1,w_1) \in \mathbb{R}^3 \) and \( P_2 = (u_2,v_2,w_2) \in \mathbb{R}^3 \). Their super – distance (see (40)) \( d_{\mathbb{R}^3}(P_2, P_3) = \sqrt{(u_2 - u_1)^2 + (v_2 - v_1)^2 + (w_2 - w_1)^2} \) does not change because
\[ \begin{align*}
\zeta &= u_2 - u_1 \\
\eta &= v_2 - v_1 \\
\xi &= w_2 - w_1
\end{align*} \] (Istennek Hála, 2014. május 11-16, 15.14.)

For example, although the point \( P_3 = (u = 0, v = 0, w = \frac{3}{2}) \) is outside our universe which has the origo \( O = (0,0,0) \), \( d_{\mathbb{R}^3}(O, P_3) = \left( \frac{3}{2} \right) \). By the super – shift transformation, (38), they have new coordinates, \( P = (\xi = 0, \eta = 0, \zeta = \frac{3}{2}) \) and \( O = (\xi = 0, \eta = 0, \zeta = \frac{3}{2}) \) but \( d_{\mathbb{R}^3}(O, P_3) = \sqrt{\left( \frac{3}{2} \right)^2 + \left( \frac{3}{2} \right)^2} = \frac{3}{2} \), again. In this case both points are visible in Fig. 14.

V. ON THE DISTANCE OF UNIVERSES

Considering two sets of points of the Multiverse \( \mathbb{W}_1 \) and \( \mathbb{W}_2 \) we define their super - distance by \( d_{\mathbb{W}_1}(\mathbb{W}_2, \mathbb{W}_3) = \inf_{P \in \mathbb{W}_1, P_1 \in \mathbb{W}_2, P_2 \in \mathbb{W}_3} d_{\mathbb{R}^3}(P_2, P_3) \). Clearly, for any subset \( \mathbb{W} \) of the Multiverse \( d_{\mathbb{W}_1}(\mathbb{W}, \mathbb{W}^3) = 0 \). Moreover, by the identity \( \mathbb{W}_1 \cap \mathbb{W}_2 = \mathbb{W}_1 \cap \mathbb{W}_2 \) we have
\[ d_{\mathbb{W}_1}(\mathbb{W}_2, \mathbb{W}_3) = \left( \frac{1}{\overbrace{\mathbb{W}_1}^{\mathbb{W}_2}} \right) \]

In the next, we classify the individual universes with respect to our universe. We say that the universe \( W_{O_0} \) (see (20)) is extremely close to our universe if \( \mathbb{W}^3 \cap W_{O_0} \neq \{ \} \), (empty set). Of course, in this case \( d_{\mathbb{R}^3}(\mathbb{W}, W_{O_0}) = 0 \). For example, the individual universe given by (21) is an extremely close to our universe. (For \( \mathbb{R}^3 \) and \( W_{O_0} \) see Fig. 7.)

We say that the universe \( W_{O_0} \) (see (20)) is neighboring our universe if \( d_{\mathbb{W}^3}(\mathbb{W}^3, W_{O_0}) = 0 \) and \( \mathbb{W}^3 \cap W_{O_0} \neq \{ \} \) (empty set). One of type of this situation is the case of the individual universe
\[ W_{O_0} = \left\{ (u,v,w) \in \mathbb{R}^3 | \left( \begin{array}{c}
1 < u < \frac{3}{2} \\
1 < v < \frac{3}{2} \\
w < \frac{3}{2} \\
\end{array} \right) \right\}, O_0 = \left( \frac{3}{2}, \frac{3}{2}, 0 \right) \in \mathbb{R}^3 \]

Our universe and \( W_{O_0} \) have the joint border point \((1,1,1)\) but it is outside both universes.

We say that the universe \( W_{O_0} \) (see (20)) is far from our universe if \( d_{\mathbb{W}^3}(\mathbb{W}^3, W_{O_0}) < 1 \), that is, the super – distance is a positive real number. For example, in the individual universe
\[ W_{O_0} = \left\{ (u,v,w) \in \mathbb{R}^3 | \left( \begin{array}{c}
0 < u < \frac{3}{2} \\
0 < v < \frac{3}{2} \\
w < \frac{3}{2} \\
\end{array} \right) \right\}, O_0 = \left( \frac{1}{7}, \frac{1}{7}, \frac{1}{7} \right) \in \mathbb{R}^3 \]

is far from our universe, because by (41) \( d_{\mathbb{W}^3}(\mathbb{W}^3, W_{O_0}) = \left( \frac{3}{7} \right) \approx 0.3613585968 \) is obtained.

The universe \( W_{O_0} \) (see (20)) is extremely far from our universe if \( d_{\mathbb{W}^3}(\mathbb{W}^3, W_{O_0}) \geq 1 \). Such kind of individual universe
\[ W_{O_0} = \left\{ (u,v,w) \in \mathbb{R}^3 | \left( \begin{array}{c}
\frac{3}{4} < u < 1 \\
\frac{3}{4} < v < 1 \\
\frac{3}{4} < w < 1 \\
\end{array} \right) \right\}, O_0 = \left( \frac{3}{4}, \frac{3}{4}, \frac{3}{4} \right) \in \mathbb{R}^3 \]

because by (41) we have that \( d_{\mathbb{W}^3}(\mathbb{W}^3, W_{O_0}) = \left( \sqrt{3} \right) > 1 \).

REFERENCES