Noise and Interference Suppression in Sonar Echoes Using the Fractional Fourier Transform

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Abstract—The Fractional Fourier Transform (FrFT) enables separation of signals from noise and interference by utilizing the entire time-frequency space. Signals are filtered by rotating to a new time axis ‘t_a’, with rotational parameter ‘a’, selected using some metric such as mean-square error (MSE) between a desired signal-of-interest (SOI) and its estimate. The FrFT has been applied to numerous problems, but it is most suited for applications such as sonar and radar, when the time-frequency distribution of the SOI and the undesired environment are different. It can greatly outperform the conventional fast Fourier Transform (FFT), which is solely a frequency domain method, as well as conventional time-based MMSE adaptive filtering (a=0). In this paper, we present a simple FrFT-based algorithm that separates sonar echoes of a desired SOI, e.g., a chirp, from the cluttered background, which could be noise or interference (i.e., another signal). We exploit the fact that we can find the best time axis ‘t_a’ in which the SOI becomes a tone, or close to it, with the FrFT, enabling easy notching (zeroing) of the clutter. By searching for the tone peak and notching everywhere except the peak, we can successfully and easily remove the clutter. This algorithm is robust because clutter typically does not correlate with the signal in the FrFT domain, and thus does not impair our ability to estimate the peaks and notch the clutter. We compute the MSE between the true transmitted signal and the received echo with and without this algorithm as a function of signal-to-noise ratio (SNR) and show that 5 dB reduction in MSE is possible with the FrFT.

Index Terms—Clutter Suppression; Fractional Fourier Transform; Sonar; Wigner Distribution.

I. INTRODUCTION

The Fractional Fourier Transform (FrFT) is a versatile tool that has been applied to problems in numerous fields, including quantum mechanics, optics [1], image processing [9], signal processing for communications ([5] and [9]), and radar [12]. The FrFT is a very useful tool for separating a signal-of-interest (SOI) from interference in a non-stationary environment [9]. The problem of separating multiple moving radar targets of differing power levels received by a monostatic radar system in clutter lends itself nicely to implementation by the FrFT because moving target echoes are chirp signals, which become tones in the proper FrFT domains, and hence can be easily separated ([12] and [13]). Previous works applying the FrFT to radar include extracting a single target in clutter using time delay correlation methods [4] and using clutter map cancellation [3]. For sonar, matched filtering is the conventional method for detecting a sonar echo and is described in many works, e.g., [7], and we seek improved methods for sonar echo estimation using the FrFT.

In this paper, we propose to use the FrFT to separate the SOI in a sonar echo from the clutter by rotating to the proper axis ‘t_a’ using rotational parameter ‘a’, in which the SOI is as close to a tone as possible. By simply searching for the maximum peak over all values of ‘a’, we easily find the correct axis. The SOI can take on numerous forms, and here we consider four types of pulses: a continuous wave (CW) tone, a chirp, a binary phase shift keying (BPSK) pulse, or a quaternary phase shift keying (QPSK) pulse. CW tones and chirps project as single tones so all but one value will be notched; BPSK/QPSK signals will not be perfect tones, but will have finite bandwidth, so the notching window will reduce. The work here differs from that in [10], which describes how chirp parameters may be estimated, but does not address how noise may be suppressed or how multiple targets could be extracted.

The paper outline is as follows: Section II briefly reviews the FrFT and its relation to the Wigner Distribution (WD), which is a useful tool for visualizing the FrFT. Section III describes the signal model and conventional detection of sonar echoes, namely the matched filter. Section IV discusses the proposed algorithm for detection using the FrFT for the four signal types considered. Section V has simulation results showing the robust performance of the proposed FrFT method. Conclusions and remarks on future work are given in Section VI.

II. BACKGROUND: FRACTIONAL FOURIER TRANSFORM (FrFT)

The discrete-time FrFT of an N × 1 vector x is

\[ X_a = F^a x, \]

(1)

where \( F^a \) is an N × N matrix whose elements are given by ([2] and [9])

\[ F^a[m,n] = \sum_{k=0}^{N-1} u_k[m] e^{-\frac{\pi}{2} a k} u_k[n], \]

(2)

and where \( u_k[m] \) and \( u_k[n] \) are the eigenvectors of the matrix S defined by [2]

\[
\begin{bmatrix}
C_0 & 1 & 0 & \ldots & 1 \\
1 & C_1 & 1 & \ldots & 0 \\
0 & 1 & C_2 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & 0 & 0 & \ldots & C_{N-1}
\end{bmatrix}
\]

(3)

And

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$$C_n = 2\cos\left(\frac{2\pi}{N} n\right) - 4.$$  

(4)

The Wigner Distribution (WD) is a time-frequency representation of a signal and may be viewed as a generalization of the Fourier Transform, which is solely the frequency representation. The WD of a signal $x(t)$ can be written as

$$W_s(t, f) = \int_{-\infty}^{\infty} x(t + \tau/2)x^*(t - \tau/2)e^{-j2\pi ft}\,d\tau.$$  

(5)

It is well-known that the projection of the WD of a signal $x(t)$ onto an axis $t_n$ gives the energy of the signal in the FrFT domain ‘a’, $|X_a(t)|^2$ (see e.g. [5] or [6]). Letting $\alpha = \pi/2$, this is written as

$$|X_a(t)|^2 = \int_{-\infty}^{\infty} W_s(t\cos(\alpha) - f\sin(\alpha), t\sin(\alpha) + f\cos(\alpha))df.$$  

(6)

In discrete time, the WD of a signal $x[n]$ is written as [8]

$$W_s[n, kf/2N] = e^{j\pi kn/N}\sum_{m=-l_1}^{l_2} x[m]x^*[n-m]e^{-j\pi km/N},$$  

(7)

where $l_1 = \max(0,n-(N-1))$ and $l_2 = \min(n,N-1)$. This particular implementation of the discrete WD is valid for non-periodic signals.

### III. SIGNAL MODEL AND CONVENTIONAL TARGET DETECTION USING MATCHED FILTERING

For a CW or chirp, we assume the transmitted signal takes the form

$$x(t) = \cos(2\pi(f_0(t-t_0) + \frac{1}{2}K(t-t_0)^2))\text{rect}\left(\frac{t-t_0}{T}\right),$$  

(8)

where $f_0 = 10$ kHz, $t_0 = 0.6$T is the chirp delay necessary to account for the rectangular pulse delay, $T = 0.006$ seconds is the length of the CW or chirp pulse; for a tone, we set $K = 0$, and for a chirp we set $K = B/T$, where $B = 4,000$ Hz is the chirp bandwidth. Rect(·) is a rectangular pulse [11].

For a BPSK or QPSK pulse, which are useful for sonar because they give better time resolution than CW tones or chirps [7], we have

$$x(t) = [I \cos(2\pi f_0(t-t_0)) + Q\sin(2\pi f_0(t-t_0))]\text{rect}\left(\frac{t-t_0}{T}\right),$$  

(9)

where I and Q are the in-phase and quadrature bits, taking on values in $\{-1,+1\}$. For BPSK, we set $Q = 0$. We further filter both pulses by a root-raised cosine (RRC) filter with roll-off $\alpha = 0.4$. The received signal is written as

$$y(t) = Ax(t-t_d) + A_jx_j(t) + \eta(t),$$  

(10)

where $A$ is the amplitude of the received pulse, taken to be $A = 1/R^2$, $t_d$ is the two way delay to the target, $t_0 = 2R/c$, $R$ is the range to the target, and $c = 1,484$ m/s is the speed of sound in water. The interfering signal, when used, is assumed to be a Gaussian pulse, given by

$$x_j(t) = \beta e^{-x((t-f_j-\phi))^2},$$  

(11)

where $\beta$ and $\phi$ are the amplitude and phase of the pulse, respectively, both uniformly distributed in $(0.5,1.5)$, $t$ is time, and $f_j$ is the sampling rate. We choose the amplitude of the interference for a specified carrier-to-interference ratio (CIR), in dB, as $A_i = 10^{-CIR/20}$. The usage of CW tones in sonar is limited; rather the use of chirps, BPSK, and QPSK, all example of pulse compression, is more common for target detection [7].

Conventional target detection collects the received signal and applies a matched filter, yielding

$$y_{MF}(t) = FFT^{-1}[FFT\{x(t)\}^*FFT\{y(t)\}].$$  

(12)

### IV. PROPOSED CLUTTER SUPPRESSION ALGORITHM USING THE FRFT

In active sonar, the signals that are transmitted by the sonar device are often tones, chirps, BPSK pulses, or QPSK pulses [7]. To describe how the FrFT is applied, we show the WD of a chirp signal $x(t)$ in Fig. 1. The figure illustrates how the FrFT may be used to transform chirp signals to tones [12] such that interference and noise can be easily removed. If we rotate to the new time axis $t_n$ and compute the energy in the FrFT, the chirp projects onto the axis as a strong tone at a particular value of time along the new time axis $t_n$, which can be found from the peak of the energy, using Eq. (6). At other values of $t_n$, there is no signal energy, and the only energy present will be from clutter. Hence, by notching everywhere but the peak, we can remove the clutter in the received signal. We then rotate the signal back to the time domain. This algorithm has been described for separating multiple echoes in chirp radar [12]. The contribution of this paper is to show how it may be applied to sonar targets, and extending its application to non-chirplike signals often used in sonar. In that regard, for BPSK and QPSK signals, we must now apply a threshold, $\gamma$, for notchig, since the signal does not project as a perfect tone, so we only notch parts of the signal less than a fraction ($\gamma$) of the peak. The algorithm can also be extended to other signal types, with the choice for $\gamma$ being the only potential modification.

![Fig. 1. Wigner Distribution of Chirp Signal $x(t)$.](image.png)

A primary difference between radar and sonar is that the Doppler observed in a received sonar echo is potentially much larger than that of a radar. Hence, it is desirable for the proposed algorithm to apply the received pulses rather than
assume knowledge of the transmitted pulses to determine the best rotational parameter ‘a’. The proposed algorithm is outlined in Table I. We search over the range of $0 < a < 2$ and find the value where the received signal peaks, indicating it is now a tone, or close to a tone. We rotate the signal using this optimum value of ‘a’, $a_{\text{opt}}$. We then notch everywhere except at the peak (for tones and chirps) or near the peak (for BPSK and QPSK pulses), accomplished using a threshold $\gamma$. The notation “$< \gamma \times \text{max} = 0$” indicates values less than $\gamma$ times the maximum value are set to zero, or notched. We let the step size for $a$, $\Delta a$, be equal to 0.01 and collect $N = 2,500$ samples of the received echo signal $y(t)$. Here, we choose $\gamma = 0.1 - 0.15$ for BPSK and QPSK signals. Note that notching everywhere but the peak is achieved when $\gamma = 1$, for tones and chirps. The FrFT output, $\hat{x}_{\text{FrFT}}(t)$, may be compared with the received signal $y(t)$ and the transmitted signal $x(t)$, which we show in the following section. We also compare the matched filter output obtained from $y(t)$, using Eq. (12), and that obtained with $\hat{x}_{\text{FrFT}}(t)$, given by

$$y_{\text{MF,FrFT}}(t) = \text{FFT}^{-1}[\text{FFT}\{x(t)\}^* \text{FFT}\{\hat{x}_{\text{FrFT}}(t)\}]$$. (13)

Note that because we utilize the entire WD space to isolate the SOI, this algorithm will be more robust than matched filtering when an unknown Doppler is present on the received signal. Also note that we suppress clutter without requiring a filter, which is a benefit to our approach because the practical usage of filters can be non-optimum [7].

**TABLE I: PROPOSED CLUTTER SUPPRESSION ALGORITHM USING THE FrFT**

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>For $a = 0 - \Delta a : 2 %$ Loop over all $a$</td>
</tr>
<tr>
<td>2.</td>
<td>$Y(a) = \text{FFT}(y(t))$; Compute FrFT of $y(t)$, given in Eq. (10)</td>
</tr>
<tr>
<td>3.</td>
<td>$Y_{\text{max}}(a) = \max(</td>
</tr>
<tr>
<td>4.</td>
<td>$a_{\text{opt}} = \arg \max_a Y_{\text{max}}(a)$; Find peak over all $a$</td>
</tr>
<tr>
<td>5.</td>
<td>$Y_{\text{peak}}(a_{\text{opt}}) = \text{FFT}(y(t))$; Rotate to signal</td>
</tr>
<tr>
<td>6.</td>
<td>$Y_{\text{peak}}(a_{\text{opt}}) &lt; \gamma \times Y_{\text{max}} = 0$; Notch out clutter</td>
</tr>
<tr>
<td>7.</td>
<td>$\gamma = 1$ for CW/chirp; $\gamma = 0.5$ for BPSK/QPSK</td>
</tr>
<tr>
<td>8.</td>
<td>$\hat{x}<em>{\text{FrFT}}(t) = \text{FFT}^{-1}[\text{FFT}{x(t)}^* \text{FFT}{\hat{x}</em>{\text{FrFT}}(t)}]$; Rotate back to $a = 0$</td>
</tr>
</tbody>
</table>

**V. SIMULATIONS**

We model four sonar signals cases: a CW tone, a chirp, a BPSK modulated pulse, and a QPSK modulated pulse. We assume the target is at a range of $R = 2.5$ meters and model a maximum range of $R_{\text{max}} = 10$ meters, so the maximum time is $t_{\text{max}} = 2R_{\text{max}}/c = 0.0135$ seconds. The signal is sampled at a sampling rate of $f_s = 200$ kHz, so the time between samples is $\Delta t = f_s^{-1} = 5 \mu s$. In the first four examples, only background noise, modeled as broadband, additive white Gaussian noise (AWGN), is assumed [7]. Plots for each case below are: Top left - the transmitted and received signals (zoomed to 100 samples to view more clearly); Top right - the FFTs of the transmitted, received, and FrFT processed signals; Bottom left - the matched filter outputs of the received and FrFT processed signals; and Bottom right - MSE between the received and FrFT processed signals. The first three plots in each example have SNR = 10 dB, but the fourth plot of MSE is over the range of SNR between -10 and 25 dB. The four cases are shown in Figs. 2 - 5. In the next four examples, we assume noise plus an interfering signal that takes on the form of a Gaussian pulse and assume that CIR = 5 dB. The results are shown in Figs. 6 - 9. Note that the targets may be mobile. We do not need to know the true chirp frequencies or the how the received chirp frequencies change due to target motion to apply this technique. From all the plots, we observe that the FrFT is able to achieve suppression of the noise at a much greater level than the matched filter. The improvement of the FrFT algorithm is significant, especially with tone and chirp pulses, even at low SNR (e.g. -5 dB); BPSK and QPSK pulses require higher SNR (e.g. 5 dB). Performance also remains robust when interference is present.

**VI. CONCLUSION**

In this paper, we apply the Fractional Fourier Transform (FrFT) to suppression of clutter in sonar signals. To apply the FrFT, we find the correct axis ‘$t_a$’ using a rotational parameter ‘a’ that rotates the signal in the Wigner plane to a tone, or as close to a tone as possible, thereby easily enabling notching of the interference and noise by setting components of the received signal to zero at values of $t_a$ that do not contain the desired tone. The FrFT improves upon the performance of a matched filter, which is unable to suppress noise, and it provides up to 5 dB improvement in error between the true transmitted signal and its estimate. The FrFT remains robust.

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in the presence of an interfering pulse or echo, even at low values of signal-to-clutter ratio.

Fig. 4. BPSK Pulse; Noise Only.

Fig. 5. QPSK Pulse; Noise Only.

Fig. 6. Tone; Noise and Gaussian Pulse Interferer, CIR = 5 dB.

Fig. 7. Chirp; Noise and Gaussian Pulse Interferer, CIR = 5 dB.

Fig. 8. BPSK Pulse; Noise and Gaussian Pulse Interferer, CIR = 5 dB.

Fig. 9. QPSK Pulse; Noise and Gaussian Pulse Interferer, CIR = 5 dB.
Future work includes applying the FrFT to separation of multiple sonar echoes, such as a target echo from reverberation echoes, using multiple FrFT domains.

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REFERENCES


Seema Sud was born in Birmingham, United Kingdom in 1971. She received the B.S. degree from the University of Maryland, College Park in 1992, the M.S. degree from George Mason University (GMU) in 1998, and the Ph.D. degree from Virginia Tech in 2002, where she was a Bradley fellow, all degrees in electrical engineering.

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Dr. Sud is a member of Tau Beta Pi, Eta Kappa Nu, Phi Kappa Phi, and Golden Key honor societies. She has received several awards for academic achievement, job performance, inventions, and teaching.