Analysis of Controlled Single-phase Full-Wave Rectifier with RL Load

Onah Aniagboso John

Abstract—Diodes are popularly used in rectifiers, which convert an ac signal into a unidirectional signal. They produce a fixed output voltage only. However, controlled switches such as thyristors are used to vary the output voltage of a converter by adjusting the delay or firing angle α of the thyristors. Phase-controlled converters are simple, efficient and less expensive. There are both single-phase and three-phase converters depending on the input supply. We also have half-wave and full-wave converters. The half-wave converter has only one polarity of output voltage and current, while for the full converter, the polarity of the output voltage can be either positive or negative. The purpose of this paper is to investigate the operation of the Single-phase full-wave rectifier. Load current for the controlled full-wave rectifier with R-L load can be either continuous or discontinuous. The paper shows how the rectifier transits from discontinuous current operation to continuous current operation.

Index Terms—Controlled Switches; Single-Phase; Delay-Angle; Rectifier; Discontinuous Current; Continuous Current.

I. INTRODUCTION

Controlled rectifiers have wide industrial applications such as dc motor speed control systems, electromechanical and electrometallurgical processes, magnet power supplies, dc transmission, portable hand tools, and uninterruptible power supplies (UPS) [1], [2]. They may be employed in closed-loop control systems, where they function as high-power operational amplifiers in which the angle α at which the thyristors are turned on is varied in response to an error signal [1]. Single-phase rectifiers are extensively applied in variable-speed drives, ranging from fractional horsepower to 15KW [3], [4]. In medium voltage ac drives, an SCR rectifier is often used as a front-end converter due to its simple structure and low manufacturing cost [5]. The polarity of its output voltage can be either positive or negative, depending on the value of α. The load current can be either continuous or discontinuous.

II. PRINCIPLE OF OPERATION

Fig. 1 [3], [6] shows the circuit of a single-phase full converter. The load consists of a series resistance, R, reactance, L, and dc voltage, Vd. Switches S1 and S2 are forward biased during the positive half-cycle of the source voltage v_s and when they are fired at α = t/ω of the delay angle), the load is connected to v_s and load current i_o flows until ωt = π + α when the switches S1 and S2 are fired during the negative half-cycle of v_s. Beyond π, S1 and S2 are still conducting though v_s has gone negative. This is due to the highly inductive load. As S1 and S2 are fired, S1 and S2 are forced to go off as a result of line or natural commutation. S1 and S2 receive firing pulses from α to π and S1 and S2 receive firing pulses from (π + α) to 2π. Trigger signal is π radians long all the time. When the pair S1, S2 conducts, v_o = v_s, and when the pair S3, S4 conducts, v_o = -v_s. When none of the switch pairs conducts, i_o = 0 and v_o = V_d [7]. When wL ≫ R the load current is essentially dc, and the load voltage V_d(t) depends solely on the timing of the semiconductor gate pulses and hence is unchanged by the presence of V_d [8]. A passive LC filter can be installed on the DC side to buffer ripple power, but due to its low resonance frequency, it increases the overall size, weight, and cost [9].

From Fig. 1, if the supply voltage is v_s = V_d sin ωt, then with the thyristors on, v_o is the voltage drop in series combination of R, L and V_d. The rectifier draws pulsed, fluctuating current i_o from the utility grid. This non-sinusoidal currents cause significant harmonic pollution, and voltage drop across the finite internal grid impedance and the voltage waveform in the vicinity becomes distorted [10]-[12]. The source voltage v_s and the switch gate pulses are shown in Fig. 2.

Published on December 7, 2018.
A. J. Onah is with Michael Okpara University, Nigeria. (e-mail: aniagbosoohnah@yahoo.com)

DOI: http://dx.doi.org/10.24018/ejers.2018.3.12.981
The average output voltage is:

\[ V_o = \frac{1}{\pi} \int_{0}^{\pi} V_m \sin \alpha \cos \omega t \, d(\omega t) = \frac{2V_m}{\pi} \cos \alpha \]  

The r.m.s. output voltage is:

\[ V_{rms} = \sqrt{\int_{0}^{\pi} V_m^2 \cos^2 \alpha \cos^2 \omega t \, d(\omega t)} = \frac{V_m}{\sqrt{2}} = V \]  

A. Continuous Current Operation

In Fourier series, the output voltage, \( v_o \) for continuous current operation can be expressed as

\[ v_o(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} \left( a_n \cos n\omega t + b_n \sin n\omega t \right) \]  

\[ a_o = \frac{1}{2\pi} \int_{0}^{2\pi} V \sin \alpha \cos \omega t \, d(\omega t) = \frac{2V_m}{\pi} \cos \alpha \] 

\[ a_n = \frac{1}{\pi} \left[ \int_{\pi/2}^{\pi} V_m \sin \alpha \cos \omega t \, d(\omega t) \right] - \int_{\pi/2}^{\pi} V_m \sin \alpha \cos \omega t \, d(\omega t) \] 

\[ a_n = \frac{2V_m}{\pi} \left[ \frac{\cos(n+1)\alpha}{n+1} - \frac{\cos(n-1)\alpha}{n-1} \right] \quad n=2,4,6, \ldots \] 

\[ b_n = \frac{1}{\pi} \left[ \int_{\pi/2}^{\pi} V_m \sin \alpha \sin \omega t \, d(\omega t) \right] - \int_{\pi/2}^{\pi} V_m \sin \alpha \sin \omega t \, d(\omega t) \] 

\[ b_n = \frac{2V_m}{\pi} \left[ \frac{\sin(n+1)\alpha}{n+1} - \frac{\sin(n-1)\alpha}{n-1} \right] \] 

\[ V_n = \sqrt{a_n^2 + b_n^2} \] 

\[ I_n = \frac{V_n}{Z_n} \]

\[ I_{rms} = \sqrt{I_o^2 + \sum_{n=2,4,6} \left( \frac{I_n}{\sqrt{2}} \right)^2} \] 

\[ z_n = \sqrt{R^2 + (n\omega L)^2} \] 

\[ \theta_n = \tan^{-1} \left( \frac{n\omega L}{R} \right) \] 

Thus:

\[ v_o = \frac{2V_m}{\pi} \cos \alpha + \sum_{n=2,4,6} \left( a_n \cos \omega t - \theta_n + \frac{b_n}{z_n} \sin (\omega t - \theta_n) \right) \] 

\[ i_o = I_o + \sum_{n=2,4,6} \left( a_n \cos (\omega t - \theta_n) + \frac{b_n}{z_n} \sin (\omega t - \theta_n) \right) \] 

\[ I_o = \frac{V_o - V_{dc}}{R} \] 

The output voltage and current waveforms are shown in Fig. 3, for \( V_{rms} = 220V, \ R = 1.5\Omega, \ L = 10mH, \ V_{dc} = 10V, \) frequency = 50Hz.

This is a typical condition of rectifier operation in continuous-current mode. Fig. 4 shows the load voltage and current waveforms when \( \alpha = 62^\circ \). Here, \( i_o = 0 \) at \( \omega t = \alpha \) and at \( \omega t = \pi + \alpha \).
As \( \alpha \) increase, the circuit tends toward inverter operation as shown in Fig. 5 and Fig. 6.

The input current can be expressed in Fourier series as:

\[
i(t) = I_{dc} + \sum_{n=1}^{\infty} \left( a_n \cos \omega t + b_n \sin \omega t \right)
\]

(18)

\[
I_{dc} = \frac{1}{2\pi} \int_{\pi}^{\pi+\alpha} I_o \cos \omega t \, dt - \int_{\pi+\alpha}^{2\pi+\alpha} I_o \cos \omega t \, dt = 0
\]

(19)

\[
a_n = \frac{1}{\pi} \int_{\pi}^{\pi+\alpha} I_o \cos \omega t \cos \omega t \, dt - \int_{\pi+\alpha}^{2\pi+\alpha} I_o \cos \omega t \cos \omega t \, dt
\]

(20)

\[
b_n = \frac{1}{\pi} \int_{\pi}^{\pi+\alpha} I_o \sin \omega t \sin \omega t \, dt - \int_{\pi+\alpha}^{2\pi+\alpha} I_o \sin \omega t \sin \omega t \, dt
\]

(21)

\[
b_n = \frac{4I_o}{n\pi} \sin n\alpha
\]

(n odd)

\[
c_n = \sqrt{a_n^2 + b_n^2}
\]

(22)

(23)

(24)

Fig. 7 shows the harmonic voltages versus delay angle \( \alpha \).
\[ I_{m} = \frac{e_{m}}{\sqrt{2}} \]  

Therefore:

\[ i_{s}(t) = \sum_{n=1,3,5}^{\infty} \frac{4I_{m}}{n\pi} [-\sin(na) \cos(n\omega t) + \cos(na) \sin(n\omega t)] \]  

\[ I_{s} = \left( \sum_{n=1,3,5,\ldots}^{\infty} I_{m}^{2} \right)^{1/2} \]  

For \( n = 1 \), \( I_{s} = I_{s1} \)

The total harmonic distortion of the input current is calculated as:

\[ THD = \frac{I_{s1}}{I_{s}} - 1 \]  

The most usual source of the supply harmonics and cause of poor power factor is the phase-controlled rectifier, whether it is applied in the motor drive, battery charger, or in uninterruptible power supplies [13]. Harmonic current-free rectifiers capable of operating at unity power factor are required as utility interfaces for these inverter-based industrial loads [14]. Large ac input filter is used to reduce high-frequency input current ripples [15].

The source voltage \( v_{s} \) and current \( i_{s} \) are shown in Fig. 9. \( I_{s1} \) is the fundamental component of \( i_{s} \).

**B. Discontinuous Current Operation**

The discontinuous-current operation is illustrated in Fig. 10. The load current becomes zero at \( \omega t = \beta \), and remains at zero until \( \omega t = \pi + \alpha \), when gate signals are applied to \( S_{1} \) and \( S_{2} \), which are then forward biased and begin to conduct. \( \beta < \pi + \alpha \).

![Fig. 9. Source voltage and current waveforms](image_url)

![Fig. 10. Discontinuous-current operation](image_url)

In Fig. 10, the waveform of \( v_{o} \) consists of three parts:

\[ v_{o} = 0; \ v_{o} = v_{s}; \ v_{o} = -v_{s} \]

**III. MODELLING**

With \( S_{1} \) and \( S_{2} \) on, the load voltage is equal to the source voltage. Thus, from Fig. 1:

\[ v_{o} = V_{m} \sin \omega t = R i_{o} + L \frac{d i_{o}}{dt} + V_{dc} \]  

\[ \frac{di_{o}}{dt} + \frac{R}{L} i_{o} = \frac{1}{L} (V_{m} \sin \omega t - V_{dc}) \]  

Solution of (30) is:

\[ i_{o} = \frac{V_{m}}{Z} \sin (\omega t - \theta) - \frac{V_{dc}}{R} e^{-\frac{R}{\alpha L} \omega t} \]  

Where \( Z = \sqrt{R^{2} + (\omega L)^{2}} \); \( \theta = \tan^{-1} \left( \frac{\omega L}{R} \right) \)

When \( \omega t = \alpha \), \( i_{o} = I_{s} \) (\( S_{1} \) and \( S_{2} \) are conducting) in steady-state condition.

\[ I_{s} = \frac{V_{m}}{Z} \sin (\alpha - \theta) - \frac{V_{dc}}{R} e^{-\frac{R}{\alpha L} \alpha} \]  

DOI: [http://dx.doi.org/10.24018/ejers.2018.3.12.981](http://dx.doi.org/10.24018/ejers.2018.3.12.981)
where $V = \frac{V_m}{R} \sin(\omega t - \theta)$.

The rms current of a switch is determined as follows:

$$I_{rms} = \frac{1}{2 \pi} \int_{0}^{\alpha} I_a d(\omega t)$$

$$I_{rms} = \left(I_R^2 + I_A^2\right)^{\frac{1}{2}} = \sqrt{2} I_R$$

The average current of a switch is found as follows:

$$I_A = \frac{1}{2} \int_{0}^{\alpha} i_a d(\omega t)$$

$$I_{dc} = I_A + I_A = 2 I_A$$

The switches may be turned on at any time that they are forward biased, at an angle $\alpha$, which should be greater than or equal to $\mu$, where $\mu = \sin^{-1}(m)$. The operation of the rectifier can be divided into three modes:

**Mode 1:** The switch current can be examined when $\alpha$ is less than $\mu$, i.e., $0 \leq \alpha \leq \mu$. Thus for $\omega t = \mu$, (34) becomes

$$I_\mu = \sin(\mu - \theta) - \frac{m}{\cos \theta} + \left[I_1 + \frac{m}{\cos \theta} - \sin(\alpha - \theta)\right] e^{\sin \theta}$$

When $\omega t = \pi + \alpha$, $i_o = I_1$ again (S1 and S4 are conducting) in steady-state condition.

$$I_1 = \frac{V_m}{Z} \sin(\pi + \alpha - \theta) - \frac{V_m}{Z} \left[I_1 + \frac{V_m}{Z} - \frac{V_m}{Z} \sin(\alpha - \theta)\right] e^{\alpha + \alpha}$$

$$I_1 = -\frac{V_m}{Z} \sin(\alpha - \theta) - \frac{V_m}{Z} \left[I_1 + \frac{V_m}{Z} - \frac{V_m}{Z} \sin(\alpha - \theta)\right] e^{\alpha + \alpha}$$

$$I_1 = -\frac{V_m}{Z} \sin(\alpha - \theta) + \frac{1}{e^{\tan \theta}} - \frac{V_m}{Z}$$

Normalize $i_o$ with $V_m/Z$ as base current:

$$i_{oN} = \sin(\omega t - \theta) - \frac{m}{\cos \theta} + \left[I_1 + \frac{m}{\cos \theta} - \sin(\alpha - \theta)\right] e^{\tan \theta}$$

$$I_{oN} = -\sin(\alpha - \theta) \left[1 + e^{-\frac{\pi}{\tan \theta}}\right] - \frac{m}{\cos \theta}$$

Where $m = \frac{Vdc}{V_m}$; $\tan \theta = \frac{\omega L}{R}$

The rms current of a switch is determined as follows:

$$I_R = \left[I_1^2 + I_A^2\right]^{\frac{1}{2}}$$

$$I_{rms} = \left(I_R^2 + I_A^2\right)^{\frac{1}{2}} = \sqrt{2} I_R$$

The average current of a switch is found as follows:

$$I_A = \frac{1}{2} \int_{0}^{\alpha} i_a d(\omega t)$$

$$I_{dc} = I_A + I_A = 2 I_A$$

The switches may be turned on at any time that they are forward biased, at an angle $\alpha$, which should be greater than or equal to $\mu$, where $\mu = \sin^{-1}(m)$. The operation of the rectifier can be divided into three modes:

**Mode 2:** for $\mu \leq \alpha \leq \pi - \mu$, the first cycle (i.e., $\omega t = \pi + \alpha$) current equation at continuous current is

$$I_{oN} = I_{oN},$$

$$I_{oN} = \sin(\pi + \alpha - \theta) - \frac{m}{\cos \theta} + \left[I_1 + \frac{m}{\cos \theta} - \sin(\alpha - \theta)\right] e^{-\frac{\pi}{\tan \theta}}$$

$$I_{oN} = -\sin(\alpha - \theta) \left[1 + e^{-\frac{\pi}{\tan \theta}}\right] - \frac{m}{\cos \theta}$$

$$I_{oN} = -\sin(\alpha - \theta) \left[1 + e^{-\frac{\pi}{\tan \theta}}\right] - \frac{m}{\cos \theta}$$

**Mode 3:** $\pi - \mu \leq \alpha \leq 2\pi - \mu$, $(\omega t = 2\pi - \mu)$, equation for current in this mode is:

$$I_A = \sin(2\pi - \mu - \theta) - \frac{m}{\cos \theta} + \left[I_1 + \frac{m}{\cos \theta} - \sin(\alpha - \theta)\right] e^{-\frac{2\pi - \mu}{\tan \theta}}$$

**Mode 3:** $\pi - \mu \leq \alpha \leq 2\pi - \mu$, $(\omega t = 2\pi - \mu)$, equation for current in this mode is:

$$I_A = \sin(2\pi - \mu - \theta) - \frac{m}{\cos \theta} + \left[I_1 + \frac{m}{\cos \theta} - \sin(\alpha - \theta)\right] e^{-\frac{2\pi - \mu}{\tan \theta}}$$
\begin{align*}
I_\mu &= -\sin(\mu - \theta) - \frac{m}{\cos \theta} \left[ -\sin(\alpha - \theta) \left[ \frac{\pi}{1 + e^{\tan \theta}} \right] - \sin(\alpha - \theta) \left[ \frac{\alpha - 2 \pi \mu}{\tan \theta} \right] e^{\frac{\theta}{\tan \theta}} \right] \\
I_\mu' &= -\sin(\mu - \theta) - \frac{m}{\cos \theta} - 2\sin(\alpha - \theta) \left[ \frac{e^\theta}{1 - e^{\tan \theta}} - \frac{1}{\tan \theta} \right] (48)
\end{align*}

where, \( x_i = \frac{\alpha - 2 \pi \mu}{\tan \theta} \)

IV. ANALYSIS

Fig. 12 is the plot of (44), (46) and (48) as \( m \) versus \( \alpha \) with \( \theta \) as parameter, where:

\[ 0 \leq \theta \leq 90^\circ ; 0 \leq \alpha \leq 180^\circ ; -1 \leq m \leq 1 \] and \( \theta = \pi/2.2, \pi/2.4, \pi/2.7, \pi/3, \pi/3.4, \pi/4, \pi/4.8, \pi/6, \pi/8, \pi/12, \pi/24, \]

Fig. 12 defines the possible and permissible conditions of operation of the rectifier. It shows a family of curves drawn for a series of values of \( \theta \). For \( m \geq 1 \), no thyristor will be turned on, and for \( m \leq -1 \), no thyristor will be turned off. Thus, an operating boundary exists at \( m = -1 \). Given the values of \( \theta \) and \( \alpha \), the curves show the values of \( m \) at which the transition from discontinuous-current operation to continuous-current operation takes place.

The operational curve for \( \theta = \pi/3 \) is shown in Fig. 13. The area A in Fig. 13b represents discontinuous-current operation in which the thyristors turn on at \( \omega t = \mu \). The area B represents continuous-current operation in which the thyristors turn on at \( \omega t = \mu \). The area C in the space bounded by the \( \sin \alpha \) curve and \( m = -1 \) represents the condition in which the thyristors will turn on at \( \omega t = \alpha \), regardless of whether the current is continuous or discontinuous.

V. CONCLUSION

The single-phase full-wave rectifier has been investigated in this article. Both the continuous-current and discontinuous-current modes of operation were analyzed. Based on a mathematical model and equations of the system, the transition of the rectifier from discontinuous current operation to continuous current operation was established. The continuous-current mode of operation, however, is of the greatest importance with regard to power circuit design and rating of circuit components. With inductive loads, this converter allows only two-quadrant operation. Thus the average output voltage could be either positive or negative according to the value of \( \alpha \). Since the total harmonic distortion (THD) of the input current is high (THD = 48.3%), an input filter inductor is normally used to decrease the current THD and also limit switching current ripple. As a rectifier, the circuit enables power to flow from the source to the load. However, the circuit can also operate as an inverter in which case power flows from the load to the ac source.

REFERENCES


DOI: http://dx.doi.org/10.24018/ejers.2018.3.12.981